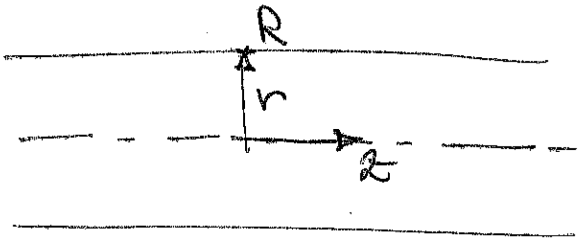
FLUSSO LAMINARE IN CONDOTTO CILINDRICO

vettore velocità in coordinate cilindriche



$$\overline{V} = u_r \hat{r} + u_\theta \hat{\theta} + u_z \hat{z}$$

Eq. N-S.

$$\frac{\partial \rho u_r}{\partial t} + \nabla \cdot (\rho u_r \overline{V}) = -\frac{\partial p}{\partial r} + \cancel{\mu}\, \nabla \cdot (\mu \nabla u_r)$$

$$\frac{\partial \rho u_z}{\partial t} + \nabla \cdot (\rho u_z \overline{V}) = -\frac{\partial p}{\partial z} + \nabla \cdot (\mu \nabla u_z)$$

Ipotesi: $u_r = 0,\ u_\theta = 0$ + flusso sviluppato:

$$\left(\frac{\partial p}{\partial z} = cost \qquad \frac{\partial u_z}{\partial z} = 0\right) \quad \begin{cases} \mu = cost \\ \rho = cost \end{cases}$$

la prima equazione ci dice:

$$\frac{\partial}{\partial t} = 0$$

$$\rightarrow \frac{\partial p}{\partial r} = 0$$

la seconda

$$\nabla \cdot (\rho u_z \overline{V}) = -\frac{\partial p}{\partial z} + \nabla \cdot (\mu \nabla u_z)$$

In coordinate cilindriche

$$\nabla f = \frac{\partial f}{\partial r} \hat{u}_r + \frac{1}{r}\frac{\partial f}{\partial \theta} \hat{\theta} + \frac{\partial f}{\partial z} \hat{z}$$

$$\nabla \cdot \overline{A} = \frac{1}{r}\frac{\partial}{\partial r}(r A_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(A_\theta) + \frac{\partial A_z}{\partial z}$$

Perciò

$$0 = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(r \mu \frac{\partial u_z}{\partial r}\right)$$

Indico $-\frac{\partial p}{\partial z} = k$

$$\mu \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial u_z}{\partial r} \right) = -k$$

Integro:

$$\mu r \frac{\partial u_z}{\partial r} = -\frac{k r^2}{2} + B$$

Integro ancora

$$u_z = -\frac{k r^2}{4\mu} + B \ln r + C$$

condizioni al contorno

$$\begin{cases} \frac{\partial u_z}{\partial r}(0) = 0 \\ u_z(R) = 0 \end{cases}$$

$$\frac{\partial u_z}{\partial r} = 0 \rightarrow B = 0$$

$$u_z(R) = -\frac{k R^2}{4\mu} + C \Rightarrow C = \frac{k R^2}{4\mu}$$

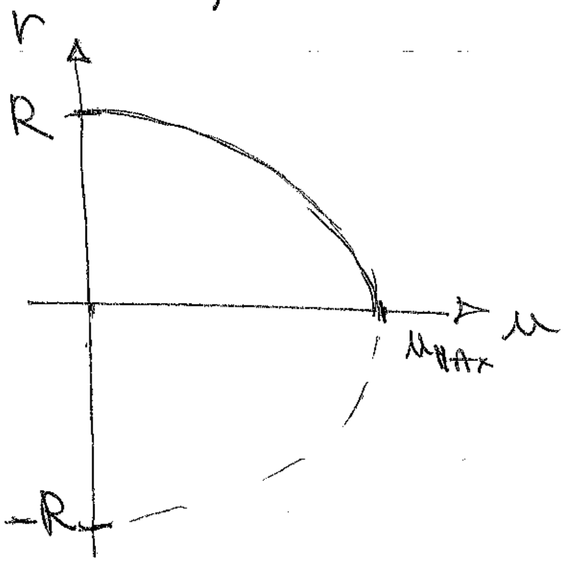
$$u_z(r) = \frac{k R^2}{4\mu} \cdot \left(1 - \frac{r^2}{R^2} \right)$$

$$\bar{u} = \frac{1}{\pi R^2} \int_0^R \frac{k R^2}{4\mu} \left(1 - \frac{r^2}{R^2} \right) 2\pi r \, dr =$$

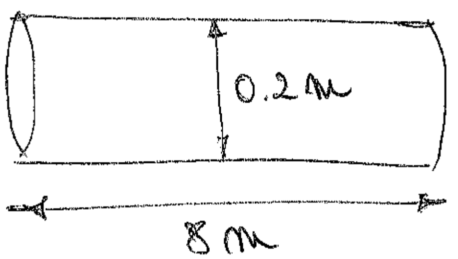
$$= \frac{k}{2\mu} \left[\frac{r^2}{2} - \frac{r^4}{4R^2} \right]_0^R = \frac{k}{2\mu} \cdot \frac{R^2}{4} = \frac{k R^2}{8\mu}$$

$$\bar{u} = \frac{u(0)}{2}$$

Perciò $$u(r) = \frac{kR^2}{4\mu}\left(1-\frac{r^2}{R^2}\right) = 2\bar{u}\left(1-\frac{r^2}{R^2}\right)$$



Esercizio

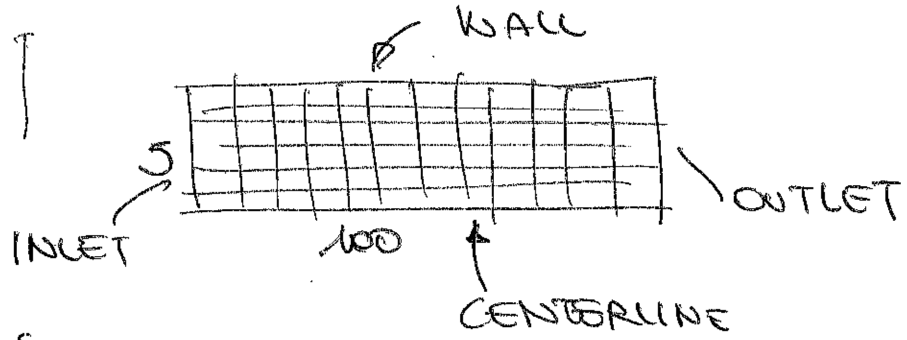


$\rho = 1\ \mathrm{kg/m^3}$ $\quad V = 1\ \mathrm{m/s}$

$\mu = 2 \cdot 10^{-3}\ \frac{\mathrm{kg}}{\mathrm{m\,s}}$

$$Re = \frac{\rho V D}{\mu} = \frac{1 \cdot \cancel{1} \cdot 0.\cancel{2}}{\cancel{2} \cdot 10^{-3}} = 100$$

griglia



- → velocity vectors
- → contours (velocity, pressure)
- → velocity profiles
- → skin friction coefficient

Skin friction coefficient

$$\frac{\pi D^{\cancel{2}}}{4}\Delta p = \tau_w \cdot \pi \cancel{D} L =$$

$$\tau_w = f \cdot \frac{1}{2}\rho V^2 = f \cdot \left(\frac{1}{2}\rho V^2\right) \cdot 4 \cdot \frac{L}{D} =$$

$$= 4f\left(\frac{1}{2}\rho V^2\right)\frac{L}{D}$$

$$\frac{\partial u}{\partial r} = 2\bar{u}\left(-\frac{2r}{R^2}\right) = \cancel{4u} - 4\frac{\bar{u}}{R} = -8\frac{\bar{u}}{D}$$

$$|\tau_w| = 8\mu\frac{\bar{u}}{D}$$

$$f = \frac{\tau_w}{\frac{1}{2}\rho \bar{u}^2} = 16\frac{\mu}{\rho \bar{u} D} = \frac{16}{Re}$$