



$I_G^D = \frac{\pi m}{\pi-1} \text{diag} \left(\frac{R^2}{4}, \frac{R^2}{4}, \frac{R^2}{2} \right)$ $I_G^T = \frac{m}{\pi-1} \text{diag} \left(\frac{R^2}{4}, \frac{R^2}{4}, \frac{R^2}{2} \right)$

$I_G^L = I_G^D - I_G^T = \frac{m R^2}{\pi-1} \text{diag} \left(\frac{6\pi-1}{24}, \frac{6\pi-1}{24}, \frac{6\pi-1}{12} \right)$

$I_G^L = I_G^L - m R^2 \text{diag} \left(\frac{1}{9}, 0, \frac{1}{9} \right) = \text{diag} \left(\frac{2\pi+1}{54(\pi-1)}, \frac{6\pi-1}{24(\pi-1)}, \frac{14\pi+1}{36(\pi-1)} \right) m R^2$

2) $T = \frac{1}{2} I_G^A \omega^2 + \frac{1}{2} I_G^L \omega^L + \frac{1}{2} m v_G^2$ $I_G^A = \frac{36 m R^2}{12} = 3 m R^2$

$\overline{OG} = \left(s \cos \theta + \frac{R}{3(\pi-1)} \sin \theta \overline{e}_1 \right) + \left(s \sin \theta - \frac{R}{3(\pi-1)} \cos \theta \right) \overline{e}_2$
 $\overline{v}_G = \left(\dot{s} \cos \theta - s \sin \theta \dot{\theta} + \frac{R}{3(\pi-1)} \cos \theta \dot{\theta} \right) \overline{e}_1 + \left(\dot{s} \sin \theta + s \cos \theta \dot{\theta} + \frac{R}{3(\pi-1)} \sin \theta \dot{\theta} \right) \overline{e}_2$
 $v_G^2 = \dot{s}^2 + s^2 \dot{\theta}^2 + \frac{R^2}{9(\pi-1)^2} \dot{\theta}^2 + \frac{R}{3(\pi-1)} \dot{s} \dot{\theta}$

$T = \frac{3}{2} m R^2 \dot{\theta}^2 + \frac{1}{2} I_G^L \dot{\theta}^2 + \frac{1}{2} m \left(\dot{s}^2 + s^2 \dot{\theta}^2 + \frac{R^2}{9(\pi-1)^2} \dot{\theta}^2 + \frac{R}{3(\pi-1)} \dot{s} \dot{\theta} \right)$ $I = \frac{16\pi+1}{36(\pi-1)} m R^2$

3) $V = m g y_G + \frac{k}{2} |oc|^L = m g \left(s \sin \theta - \frac{R}{3(\pi-1)} \cos \theta \right) + \frac{3(\pi-1)}{4R} m g s^2$

$\frac{\partial V}{\partial s} = m g \sin \theta + \frac{3(\pi-1)}{2R} m g s$ $s = -\frac{2R}{3(\pi-1)} \sin \theta$

$\frac{\partial V}{\partial \theta} = m g \left(s \cos \theta + \frac{R}{3(\pi-1)} \sin \theta \right) \rightarrow \sin \theta (-2 \cos \theta + 1) = 0$

$\sin \theta = 0 \quad \theta < 0 \quad s < 0 \quad \cos \theta = \frac{1}{2} \quad \theta = \frac{\pi}{3} \quad s = -\frac{\sqrt{3}R}{3(\pi-1)}$
 $\theta = \pi \quad s < 0 \quad \theta = -\frac{\pi}{3} \quad s = \frac{\sqrt{3}R}{3(\pi-1)}$

$\frac{\partial^2 V}{\partial s^2} = \frac{3(\pi-1)m g}{2R} > 0$ $\frac{\partial^2 V}{\partial \theta^2} = m g \left(-s \sin \theta + \frac{R}{3(\pi-1)} \cos \theta \right)$ $\frac{\partial^2 V}{\partial \theta \partial s} = m g \cos \theta$

$2R \sin \theta = 0 \quad k < 0 \quad \text{INST.} \quad 2R \cos \theta = \frac{1}{2} \quad 2R > 0 \quad \text{STAB.}$

4) $T \sim \frac{m}{2} \left[\dot{s}^2 + \frac{R}{3(\pi-1)} \dot{s} \dot{\theta} + \left(3R^2 + \frac{I}{m} + \frac{4R^2}{9(\pi-1)^2} \right) \dot{\theta}^2 \right]$

$A = \begin{pmatrix} m & \frac{mR}{6(\pi-1)} \\ \frac{mR}{6(\pi-1)} & I + 3mR^2 + \frac{4mR^2}{9(\pi-1)^2} \end{pmatrix}$ $C = \begin{pmatrix} \frac{3(\pi-1)m g}{2R} & \frac{m g}{2} \\ \frac{m g}{2} & \frac{2m g R}{3(\pi-1)} \end{pmatrix}$ $\det(A-C) = 0$

5) $L_0 = I_G(\overline{\omega}) + \overline{r}_{G0} \times \overline{v}_G + I_G(\overline{\omega})$