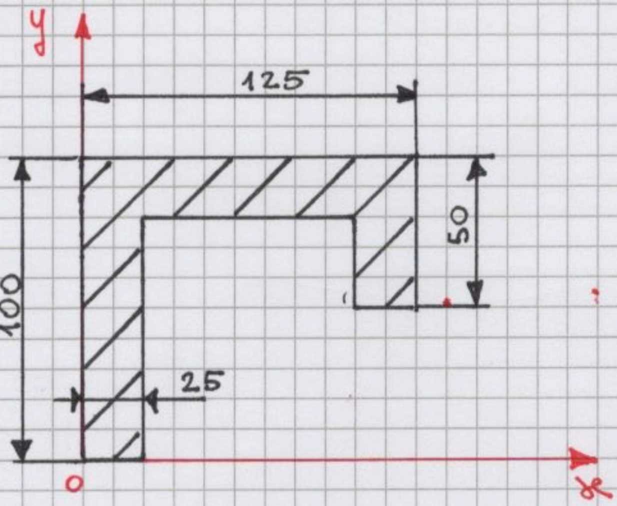
GEOMETRIA DELLE AREE 12/11/2018



CALCOLO DEL BARICENTRO

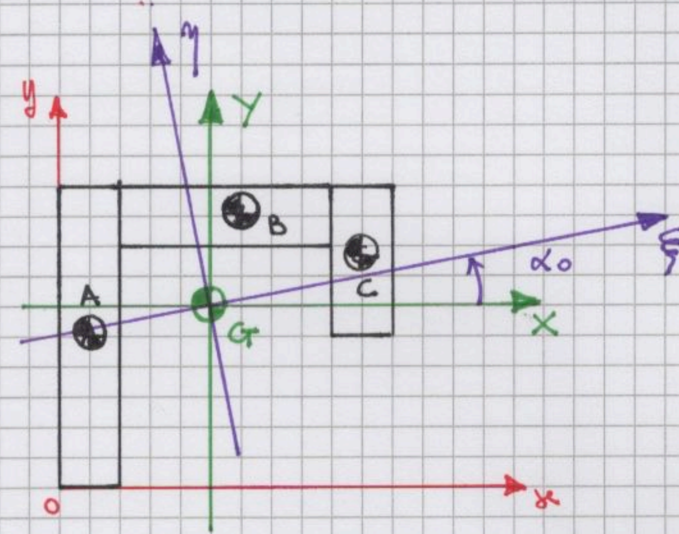
Sistema di riferimento $\{O\, xy\}$

COORDINATE BARICENTRICHE

$$A \begin{cases} x_{GA} = 12.5 \text{ mm} \\ y_{GA} = 50 \text{ mm} \end{cases}$$

$$B \begin{cases} x_{GB} = 62.5 \text{ mm} \\ y_{GB} = 87.5 \text{ mm} \end{cases}$$

$$C \begin{cases} x_{GC} = 112.5 \text{ mm} \\ y_{GC} = 75 \text{ mm} \end{cases}$$



$A_A = 100 \cdot 25 = 2500 \text{ mm}^2$

$A_B = 75 \cdot 25 = 1875 \text{ mm}^2$

$A_C = 50 \cdot 25 = 1250 \text{ mm}^2$

MOMENTI STATICI

$$S_x = A_A \cdot y_{GA} + A_B\, y_{GB} + A_C\, y_{GC} =$$

$$= (2500)\,50 + (1875)\,87.5 + (1250)\,75 = 382\,812.5 \text{ mm}^3$$

$$S_y = A_A\, x_{GA} + A_B\, x_{GB} + A_C\, x_{GC} =$$

$$= (2500)\,12.5 + (1875)\,62.5 + (1250)\,112.5 = 289\,062.5 \text{ mm}^3$$

$$x_G = \frac{S_y}{A_{TOT}} = \frac{289\,062.5}{5625} = 51.3889 \text{ mm}$$

$$y_G = \frac{S_x}{A_{TOT}} = \frac{382\,812.5}{5625} = 68.0556 \text{ mm}$$

SISTEMA BARICENTRICO

Una volta determinate le coordinate del baricentro della sezione è comodo esprimere le coordinate dei baricentri dei rettangoli A, B e C nel sistema baricentrico $\{G\ XY\}$

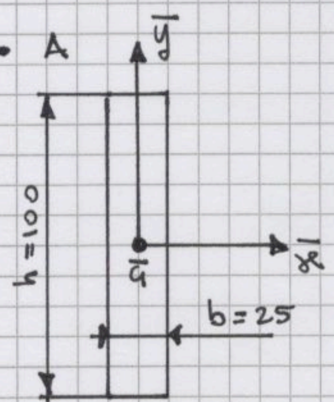
$$A\begin{cases} X_{GA} = x_{GA} - x_G = 12.5 - 51.3889 = -38.8889\ mm \\ Y_{GA} = y_{GA} - y_G = 50 - 68.0556 = -18.0556\ mm \end{cases}$$

$$B\begin{cases} X_{GB} = x_{GB} - x_G = 11.1111\ mm \\ Y_{GB} = y_{GB} - y_G = 19.4444\ mm \end{cases}$$

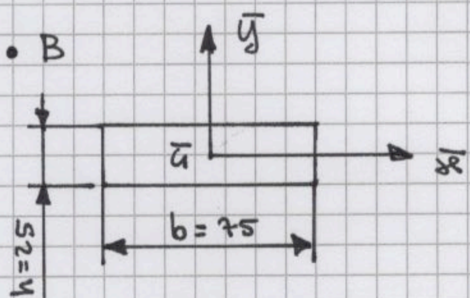
$$C\begin{cases} X_{GC} = x_{GC} - x_G = 61.1111\ mm \\ Y_{GC} = y_{GC} - y_G = 6.9444\ mm \end{cases}$$

CALCOLO DEI MOMENTI D'INERZIA.

• A



• B



$$I_{x_A} = \frac{bh^3}{12} = \frac{25 \cdot 100^3}{12} = 2'083'333,3333\ mm^4$$

$$I_{y_A} = \frac{hb^3}{12} = \frac{100 \cdot 25^3}{12} = 130\ 208.3333\ mm^4$$

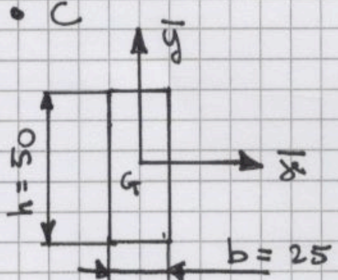
$$I_{xy_A} = \emptyset$$

$$I_{x_B} = \frac{bh^3}{12} = 97656.25\ mm^4$$

$$I_{y_B} = \frac{hb^3}{12} = 878\ 906.25\ mm^4$$

$$I_{xy_B} = \emptyset$$

• C



$$I_{x_C} = 260\ 416,6667\ mm^4$$

$$I_{y_C} = 65104.1667\ mm^4$$

$$I_{xy_C} = \emptyset$$

CALCOLO DEI MOMENTI D'INERZIA

$$I_x = I_{xA} + A_A (y_{GA})^2 + I_{xB} + A_B (y_{GB})^2 + I_{xC} + A_C (y_{GC})^2 =$$

$$= I_{xA} + 2500(-18.0556)^2 + I_{xB} + 1875(19.4444)^2 + I_{xC} + 1250(6.9444)^2 =$$

$$= 4'025'607,6389 \ mm^4$$

$$I_y = I_{y_A} + A_A (x_{GA})^2 + I_{y_B} + A_B (x_{GB})^2 + I_{y_C} + A_C (x_{GC})^2 =$$

$$= I_{y_A} + 2500(-38.8889)^2 + I_{y_B} + 1875(11.1111)^2 + I_{y_C} + 1250(61.1111)^2 =$$

$$= 9'754'774,3056 \ mm^4$$

$$I_{xy} = \underbrace{I_{xy_A}}_{\emptyset} + A_A (x_{GA})(y_{GA}) + \underbrace{I_{xy_B}}_{\emptyset} + A_B (x_{GB})(y_{GB}) + \underbrace{I_{xy_C}}_{\emptyset} + A_C (x_{GC})(y_{GC}) =$$

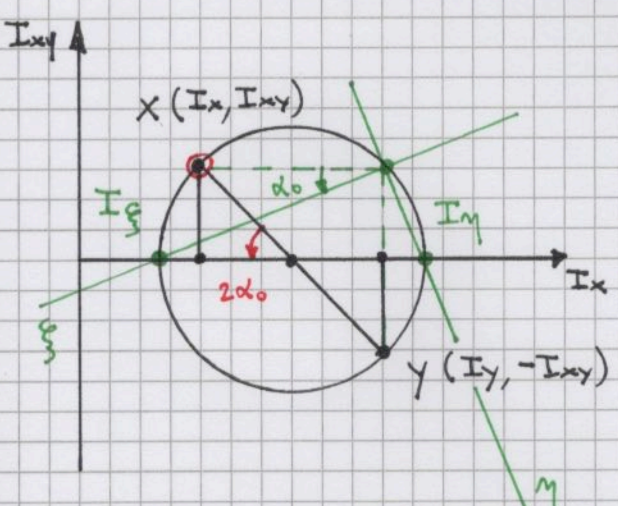
$$= 2500 \underbrace{(-18.0556)}_{y_{GA}} \underbrace{(-38.8889)}_{x_{GA}} + 1875 \underbrace{(19.4444)}_{y_{GB}} \underbrace{(11.1111)}_{x_{GB}} + 1250 \underbrace{(6.9444)}_{y_{GC}} \underbrace{(61.1111)}_{x_{GC}} =$$

$$= 2'690'972,2222 \ mm^4$$

CERCHIO DI MOHR $\quad I_y > I_x \quad I_{xy} > \emptyset$

$$C = \frac{I_x + I_y}{2} = 6'890'190,9722 \ mm^4$$

$$R = \frac{1}{2}\sqrt{(I_y - I_x)^2 + (2 I_{xy})^2} = 3'930'288,6884 \ mm^4$$



$$\begin{matrix} I_\xi \\ I_\eta \end{matrix} = C \mp R = \begin{cases} \text{MIN} & 2'959'902,2838 \ mm^4 \\ \text{MAX} & 10'820'479,6606 \ mm^4 \end{cases}$$

$$\alpha_0 = \frac{1}{2} \operatorname{atan}\left(\frac{2 I_{xy}}{I_y - I_x}\right) = 21.6050°$$