Boundary integral formulation of interfacial cracks in thermodiffusive bimaterials

Lorenzo Morini

Department of Civil, Environmental and Mechanical Engineering, University of Trento, Italy,
IMT insitute for Advanced Studies, Lucca, Italy,
E-mails: lorenzo.morini@unitn.it,

A general approach for studying cracks propagation at the interface between dissimilar elastic materials in presence of thermal conduction and mass diffusion is proposed. A generalized form of the reciprocity theorem is used in order to formulate the problem in terms of boundary integral equations, and additional volume integral terms due to thermodiffusive effects are reduced to surface integrals by means of an exact transformation. Applying the original method recently developed by the authors [5], explicit symmetric and skew-symmetric weight function matrices are used as auxiliary fields into the derived Betti's identity [4]. A system of integral equations relating the applied loading, the temperature and mass concentration fields on the fracture surfaces and the resulting crack opening is derived. The proposed approach is particularly appropriate for studying crack propagation at the interface between different components in lithium ions batteries and solid oxide fuel cells (SOFCs), which in standard operational conditions are subjected to extreme environments and severe thermomechanical and diffusive residual stresses allowing to structural damages and fracture processes [1, 2]. The obtained integral identities are then used for solving some examples of fracture problems where the contribution of the temperature, mass concentration, heat and mass fluxes on the resulting crack opening and traction ahead of the crack tip can be calculated in closed form. Exact expressions for the thermal and diffusive contributions to the stress intensity factors are also derived in some illustrative cases.

References

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