1) $T = \frac{1}{2} \underset{O}{I}^{an} \omega_{an}^2 + \frac{1}{2} m v_G^2 + \frac{1}{2} \underset{G}{I}^{as} \omega_{as}^2$

$\overline{OG} = \overline{OC} + \overline{CG} = R\left(\sin\theta + \frac{1}{2}\sin\varphi\right)\bar{e}_1 - R\left(\cos\theta + \frac{1}{2}\cos\varphi\right)\bar{e}_2$

$\bar{v}_G = R\left(\cos\theta\,\dot\theta + \frac{1}{2}\cos\varphi\,\dot\varphi\right)\bar{e}_1 + R\left(\sin\theta\,\dot\theta + \frac{1}{2}\sin\varphi\,\dot\varphi\right)\bar{e}_2$

$v_G^2 = R^2\left(\dot\theta^2 + \frac{1}{4}\dot\varphi^2 + \cos(\theta-\varphi)\dot\varphi\dot\theta\right)$

$\underset{O}{I}^{an} = mR^2 + mR^2 = 2mR^2$ $\quad \underset{G}{I}^{as} = \frac{1}{12} m(\sqrt{3}R)^2 = \frac{1}{4} mR^2$

$T = \frac{3}{2} mR^2\dot\theta^2 + \frac{3}{8} mR^2\dot\varphi^2 + \frac{1}{2} mR^2\cos(\theta-\varphi)\dot\theta\dot\varphi$

$V = mg\,y_C + mg\,y_G - \bar{F}\cdot\overline{OH} = -mgR\left(\cos\theta + \frac{1}{2}\cos\varphi\right) - mgR\cos\theta$

$-\frac{2\sqrt{3}}{3} mgR\left(\sin\theta + \frac{1}{2}\sin\varphi - \frac{\sqrt{3}}{4}\cos\varphi\right) = -mgR\left(2\cos\theta + \frac{2\sqrt{3}}{3}\sin\theta + \frac{\sqrt{3}}{3}\sin\varphi\right)$

2) $\frac{\partial V}{\partial\theta} = 2mgR\left(\sin\theta - \frac{\sqrt{3}}{3}\cos\theta\right) = 0$ $\quad \tan\theta = \frac{\sqrt{3}}{3}$ $\quad \theta = \frac{\pi}{6}, \frac{7\pi}{6}$

$\frac{\partial V}{\partial\varphi} = -\frac{\sqrt{3}}{3} mgR\cos\varphi = 0$ $\quad \varphi = \pm\frac{\pi}{2}$

$\frac{\partial^2 V}{\partial\theta^2} = 2mgR\left(\cos\theta + \frac{\sqrt{3}}{3}\sin\theta\right)$ $\quad \frac{\partial^2 V}{\partial\varphi^2} = +\frac{\sqrt{3}}{3} mgR\sin\varphi$ $\quad \frac{\partial^2 V}{\partial\varphi\partial\theta} = 0$

equilibrio stabile se $\frac{\partial^2 V}{\partial\theta^2} > 0$, $\frac{\partial^2 V}{\partial\varphi^2} > 0$ $\quad \theta = \frac{\pi}{6}$ $\quad \varphi = +\frac{\pi}{2}$

3) $T \sim \frac{1}{2} mR^2\left(3\dot\theta^2 + \frac{3}{4}\dot\varphi^2 + \frac{1}{2}\dot\varphi\dot\theta\right)$

$A = \begin{pmatrix} 3mR^2 & \frac{mR^2}{4} \\ \frac{mR^2}{4} & \frac{3}{4}mR^2 \end{pmatrix}$ $\quad C = \begin{pmatrix} \frac{2}{\sqrt{3}} mgR & 0 \\ 0 & \frac{mgR}{\sqrt{3}} \end{pmatrix}$ $\quad \det|C - \lambda^2 A| = 0$

4) $\underset{O}{L} = \underset{O}{I}^{an}(\omega_{an}) + \bar{r}_G \times \bar{v}_G + \underset{G}{I}^{as}(\omega_{as}) = \bar{e}_3\Big[2mR^2\dot\theta + \frac{1}{4} mR^2\dot\varphi$

$+ mR^2\Big(\left(\sin\theta + \frac{1}{2}\sin\varphi\right)\left(\cos\theta\,\dot\theta + \frac{1}{2}\cos\varphi\,\dot\varphi\right) + \left(\cos\theta + \frac{1}{2}\cos\varphi\right)\left(\sin\theta\,\dot\theta + \frac{1}{2}\sin\varphi\,\dot\varphi\right)$

$= \Big[2mR^2\dot\theta + \frac{1}{4} mR^2\dot\varphi + \Big(2\sin\theta\cos\theta\,\dot\theta + \frac{1}{4}\sin\varphi\cos\varphi\,\dot\varphi + \cos(\theta-\varphi)(\dot\varphi + \dot\theta)\Big]\bar{e}_3$