$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$

$\overline{v_G} = (s-3\ell)(\cos\theta\, \bar{e}_1 + \sin\theta\, \bar{e}_2)$

$\overline{v_G} = [\dot{s}\cos\theta - (s-3\ell)\sin\theta\,\dot{\theta}]\bar{e}_1 + [\dot{s}\sin\theta + (s-3\ell)\cos\theta\,\dot{\theta}]\bar{e}_2 \qquad v_G^2 = \dot{s}^2 + (s-3\ell)^2\dot{\theta}^2$

$I_G = \frac{1}{12} m\, 36\ell^2 = 3m\ell^2 \qquad \omega = \dot{\theta}$

$T = \frac{1}{2} m(\dot{s}^2 + (s-3\ell)^2\dot{\theta}^2) + \frac{3}{2} m\ell^2\dot{\theta}^2 = \frac{1}{2} m\dot{s}^2 + \frac{1}{2} m\,(s^2 - 6\ell s + 12\ell^2)\dot{\theta}^2$

$V_F = -4mg\ell \int_{s_0}^{s} \frac{ds'}{s'} - 4mg\ell \int_{s_0}^{6\ell - s} \frac{ds'}{s'} = -4mg\ell\,[\log s + \log(6\ell - s)] + \text{cost}$

$V_P = mg\, y_G = mg(s-3\ell)\sin\theta$

$V = V_F + V_P = -4mg\ell\,[\log s + \log(6\ell - s)] + mg(s-3\ell)\sin\theta$

$L = \frac{1}{2} m\dot{s}^2 + \frac{1}{2} m(s^2 - 6\ell s + 12\ell^2)\dot{\theta}^2 - mg(s-3\ell)\sin\theta + 4mg\ell(\log s + \log(6\ell - s))$

$m\ddot{s} - m(s-3\ell)\dot{\theta}^2 - \frac{4mg\ell}{s} + \frac{4mg\ell}{6\ell - s} + mg\sin\theta = 0$

$m(s^2 - 6\ell s + 12\ell^2)\ddot{\theta} + 2m(s-3\ell)\dot{s}\dot{\theta} + mg(s-3\ell)\cos\theta = 0$

$\frac{\partial V}{\partial \theta} = mg(s-3\ell)\cos\theta = 0 \qquad \frac{\partial V}{\partial s} = mg\left(-\frac{4\ell}{s} + \frac{4\ell}{6\ell - s} + \sin\theta\right) = 0$

Sse: $s = 3\ell$ o $\cos\theta = 0$ (con $0 < s < 6\ell$)

1) $s = 3\ell, \theta = 0$ 2) $s = 3\ell, \theta = \pi$ 3) $s = 2\ell, \theta = \frac{\pi}{2}$ 4) $s = 12\ell, \theta = \frac{\pi}{2}$ non accettabile

5) $s = 4\ell, \theta = -\frac{\pi}{2}$ 6) $s = -6\ell, \theta = -\frac{\pi}{2}$ non accettabile

$\frac{\partial^2 V}{\partial \theta^2} = -mg(s-3\ell)\sin\theta \qquad \frac{\partial^2 V}{\partial s \partial \theta} = mg\cos\theta \qquad \frac{\partial^2 V}{\partial s^2} = 4mg\ell\left(\frac{1}{s^2} + \frac{1}{(6\ell - s)^2}\right) > 0$

$V_{ss} > 0$ sempre $\qquad H = m^2 g^2 \left[\left(\frac{1}{s^2} + \frac{1}{(6\ell - s)^2}\right)(s-3\ell)\sin\theta + \cos^2\theta\right]$

$H > 0$ per $s = 2\ell, \theta = \frac{\pi}{2}$ o $s = 4\ell, \theta = -\frac{\pi}{2}$

In $s = 2\ell, \theta = \frac{\pi}{2} \qquad T \sim \frac{m}{2}(\dot{s}^2 + 4\ell^2\dot{\theta}^2)$

$A = \begin{pmatrix} m & 0 \\ 0 & 4m\ell^2 \end{pmatrix} \qquad C = \begin{pmatrix} \frac{5}{4} mg/\ell & 0 \\ 0 & mg\ell \end{pmatrix} \qquad \lambda_1 = \frac{5}{4}\frac{g}{\ell} \qquad \lambda_2 = \frac{1}{4}\frac{g}{\ell}$

moti traslatori se $\dot{\theta} = \theta = 0$. Dalle eq. di Lagrange segue $\cos\theta = 0$

e $m\ddot{s} - \frac{4mg\ell}{s} + \frac{4mg\ell}{6\ell - s} \pm mg = 0$ OK