$$T=\frac{1}{2}I_O^{(\ell)}\dot\theta^2+\frac{1}{2}m v_G^2+\frac{1}{2}I_G^{(a)}\dot\theta^2$$

$$I_O^{(\ell)}=\frac{1}{12}m(\ell^2+3\ell^2)+m\left(\frac{\ell^2}{2}+3\frac{\ell^2}{2}\right)=\frac{4}{3}m\ell^2 \qquad I_G^{(a)}=\frac{1}{12}m\ell^2$$

$$\bar r_G=\left[\left(\frac{\sqrt3}{2}\ell-s\right)\operatorname{sen}\theta+\frac{\ell}{2}\cos\theta\right]\bar e_1+\left[\left(\frac{\sqrt3}{2}\ell-s\right)\cos\theta-\frac{\ell}{2}\operatorname{sen}\theta\right]\bar e_2$$

$$\bar v_G=\left[\left(\frac{\sqrt3}{2}\ell-s\right)\cos\theta\,\dot\theta-\frac{\ell}{2}\operatorname{sen}\theta\,\dot\theta-\dot s\operatorname{sen}\theta\right]\bar e_1+\left[-\left(\frac{\sqrt3}{2}\ell-s\right)\operatorname{sen}\theta\,\dot\theta-\frac{\ell}{2}\cos\theta\,\dot\theta-\dot s\cos\theta\right]\bar e_2$$

$$v_G^2=(\ell^2-\sqrt3\ell s+s^2)\dot\theta^2+\dot s^2+\ell\dot s\dot\theta$$

$$T=\frac{m}{2}\left[\frac{4}{3}\ell^2\dot\theta^2+\frac{1}{12}\ell^2\dot\theta^2+(\ell^2-\sqrt3\ell s+s^2)\dot\theta^2+\dot s^2+\ell\dot s\dot\theta\right]=\frac{m}{2}\left[\left(\frac{29}{12}\ell^2-\sqrt3\ell s+s^2\right)\dot\theta^2+\dot s^2+\ell\dot s\dot\theta\right]$$

$$V=mgy_F+mgy_G+\frac{k}{2}|QC|^2+\frac{k}{2}|HH|^2$$

$$y_F=\frac{\sqrt3}{2}\ell\cos\theta-\frac{\ell}{2}\operatorname{sen}\theta \qquad |QC|^2=6\ell^2(1-\cos\theta)$$

$$V=mg\ell(\sqrt3\cos\theta-\operatorname{sen}\theta)-mgs\cos\theta-\frac{\sqrt3}{6}\frac{mg}{\ell}6\ell^2\cos\theta+\frac{\sqrt3}{4}\frac{mg}{\ell}s^2+\text{cost}$$

$$=\frac{\sqrt3}{4}mg\frac{s^2}{\ell}-mg\ell\operatorname{sen}\theta-mgs\cos\theta$$

$$\frac{\partial V}{\partial s}=\frac{\sqrt3}{2}mg\frac{s}{\ell}-mg\cos\theta=0 \quad\rightarrow\quad s=\frac{2}{\sqrt3}\ell\cos\theta$$

$$\frac{\partial V}{\partial\theta}=mg(s\operatorname{sen}\theta-\ell\cos\theta)=0 \quad\rightarrow\quad \cos\theta\left(\frac{2}{\sqrt3}\operatorname{sen}\theta-1\right)=0 \quad < \begin{matrix}\cos\theta=0\\ \operatorname{sen}\theta=\frac{\sqrt3}{2}\end{matrix}$$

$$\theta=\pm\frac{\pi}{2}\ \ s=0\ ;\quad \theta=\frac{\pi}{3},\ s=\frac{\ell}{\sqrt3}\ ;\quad \theta=\frac{2\pi}{3},\ s=-\frac{\ell}{\sqrt3}$$

$$\frac{\partial^2V}{\partial s^2}=\frac{\sqrt3}{2}\frac{mg}{\ell} \qquad \frac{\partial^2V}{\partial\theta^2}=mg(s\cos\theta+\ell\operatorname{sen}\theta) \qquad \frac{\partial^2V}{\partial s\partial\theta}=mg\operatorname{sen}\theta$$

$$\theta=\pm\frac{\pi}{2},\ s=0 \quad H=\begin{pmatrix}\frac{\sqrt3}{2}\frac{mg}{\ell} & \pm mg\\ \pm mg & \pm\ell\end{pmatrix}=\left(\pm\frac{\sqrt3}{2}-1\right)mg<0 \quad \text{instabile}$$

$$\begin{matrix}\theta=\frac{\pi}{3} & s=\frac{\ell}{\sqrt3}\\ \theta=\frac{2\pi}{3} & s=-\frac{\ell}{\sqrt3}\end{matrix} \quad H=\begin{pmatrix}\frac{\sqrt3}{2}\frac{mg}{\ell} & \frac{\sqrt3}{2}mg\\ \frac{\sqrt3}{2}mg & \frac{3}{\sqrt3}mg\ell\end{pmatrix}=\frac{mg\ell}{4}>0 \quad \text{stabile}$$

$$T\sim\frac{m}{2}\left(\dot s^2+\frac{21}{12}\ell^2\dot\theta^2+\ell\dot\theta\dot s\right) \qquad A=\begin{pmatrix}m & \frac{m}{2}\ell\\ \frac{m}{2}\ell & \frac{21}{12}m\ell^2\end{pmatrix} \qquad C=\begin{pmatrix}\frac{\sqrt3}{2}\frac{mg}{\ell} & \frac{\sqrt3}{2}mg\\ \frac{\sqrt3}{2}mg & \frac{2}{\sqrt3}mg\ell\end{pmatrix}$$

$$\det(C-\lambda A)=0$$

$$\Gamma_O=\bar r_G\times m\bar v_G+I_G(\bar\omega)$$