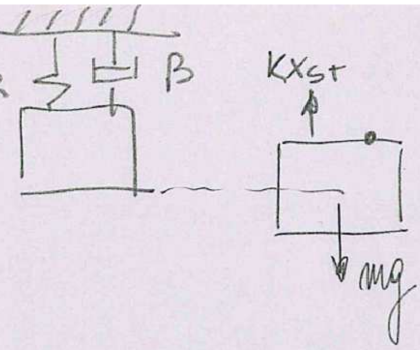
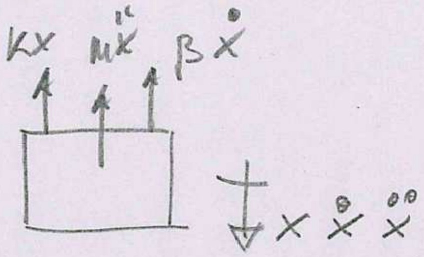
FONDAMENTI DI MECCANICA E BIOMECCANICA [IN/0165]

Lezione del 30 novembre e 01 dicembre 2017.

Titolo:
Oscillazioni ad un grado di libertà.

Contenuti:
Oscillazioni ad un grado di libertà nell'intorno della configurazione di equilibrio statico.
Oscillazioni libere smorzate e non smorzate.
Oscillazioni forzate.
Trasmissibilità.
Risonanza.
Applicazioni.



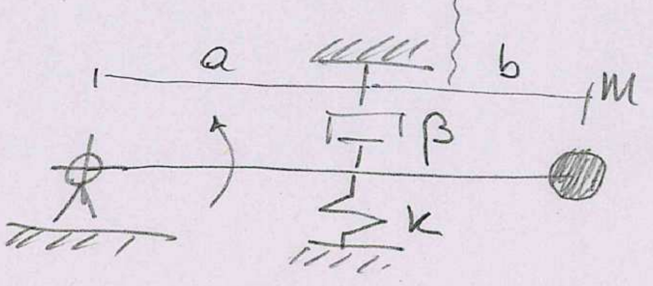


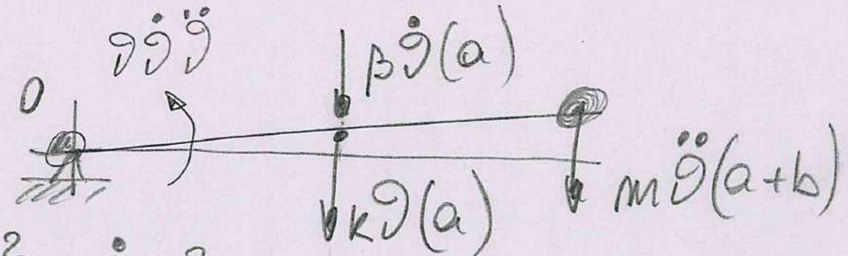
$$\ddot{x} + \left(2\zeta\omega_n\right)\dot{x} + \left(\omega_n^2\right)x = 0$$

$$\zeta = \frac{\beta}{2m\omega_n} \qquad \omega_n = \sqrt{\frac{k}{m}}$$

$$m\ddot{x} + \beta\dot{x} + kx = 0$$

$$\ddot{x} + \left(\frac{\beta}{m}\right)\dot{x} + \left(\frac{k}{m}\right)x$$

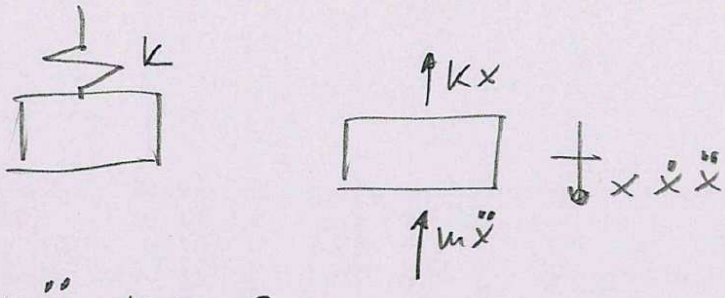




$$\circlearrowleft 0) \quad m\ddot{\vartheta}(a+b)^2 + \beta\dot{\vartheta}a^2 + k\vartheta a^2 = 0$$

$$\ddot{\vartheta} + \left(\frac{\beta a^2}{m(a+b)^2}\right)\dot{\vartheta} + \left(\frac{k a^2}{m(a+b)^2}\right)\vartheta = 0$$

$$\omega_n = \sqrt{\frac{k a^2}{m(a+b)^2}}$$



$$m\ddot{x} + Kx = 0$$

$$\ddot{x} + \frac{k}{m}x = 0 \qquad \ddot{x} + \omega_n^2 x = 0$$

- $x = A\cos\omega_n t + B\sin\omega_n t$

$$x = e^{\lambda t} \quad \dot{x} = \lambda e^{\lambda t} \quad \ddot{x}\,\lambda^2 e^{\lambda t}$$

$$\lambda^2 + \omega_n^2 = 0 \qquad \lambda_{1,2} = \pm i\omega_n$$

$$x = a\,e^{i\omega_n t} + b\,e^{-i\omega_n t}$$

EULERO

$$e^{\pm i\alpha} = \cos\alpha \pm i\sin\alpha$$

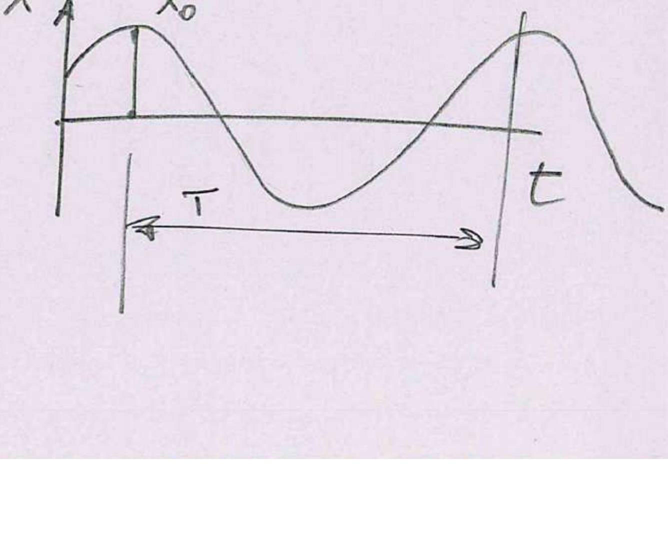
- $x = \underbrace{(a+b)}_{A}\cos\omega_n t + \underbrace{i(a-b)}_{B}\sin\omega_n t$

$$X = X_0 \sin(\omega_n t + \varphi_0)$$

$$A = x_0 \sin\varphi_0 \; ; \quad B = x_0\cos\varphi_0$$

$$T = \frac{2\pi}{\omega_n} \qquad f = \frac{1}{T}$$

$$\omega_n = \sqrt{\frac{k}{m}}$$



$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$$

$$\zeta = \frac{\beta}{2m\omega_n} \qquad \omega_n = \sqrt{\frac{k}{m}} \qquad \zeta = \frac{\beta}{\beta_{CR}}$$

$$x = e^{\lambda t} \qquad \dot{x} \qquad \ddot{x}$$

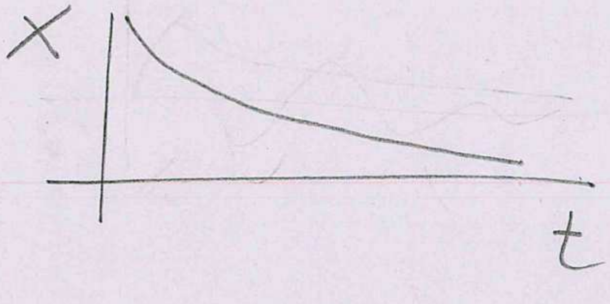
$$\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0$$

$$\lambda_{1,2} = -\zeta\omega_n \pm \sqrt{\zeta^2\omega_n^2 - \omega_n^2} = \omega_n\left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)$$

$$x = a e^{\lambda_1 t} + b e^{\lambda_2 t}$$

$\zeta > 1 \quad \lambda_{1,2} \in \mathbb{R}$ DISTINTI

$$x = \underbrace{e^{-\zeta\omega_n t}}_{A} \underbrace{\left(\underbrace{a e^{(\sqrt{\zeta^2-1})\omega_n t}}_{C} + \underbrace{b e^{(-\sqrt{\zeta^2-1}\,\omega_n t)}}_{D}\right)}_{B}$$



$$\lim_{t \to \infty} x(t) = 0$$

$\zeta < 1$

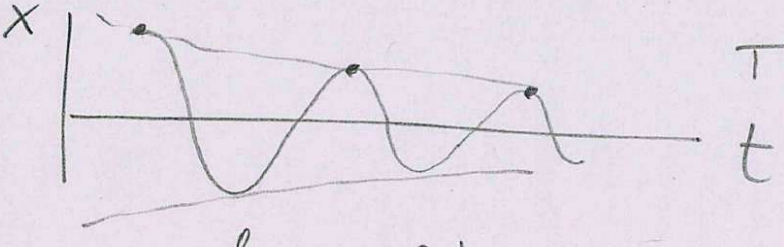
$$\lambda_{1,2} = -\zeta\omega_n \pm i\omega_n\sqrt{1-\zeta^2}$$

$$x = a e^{\lambda_1 t} + b e^{\lambda_2 t}$$

$$x = x_0 e^{-\zeta\omega_n t} \cos(\omega_s t + \varphi_0)$$

$$\omega_s = \omega_n\sqrt{1-\zeta^2} < \omega_n$$

ω_s PULS. PROPRIA SIST. SMORZ.



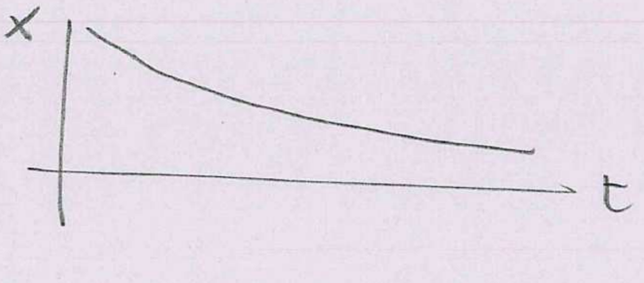
$$T = \frac{2\pi}{\omega_s} \quad f = \frac{1}{T}$$

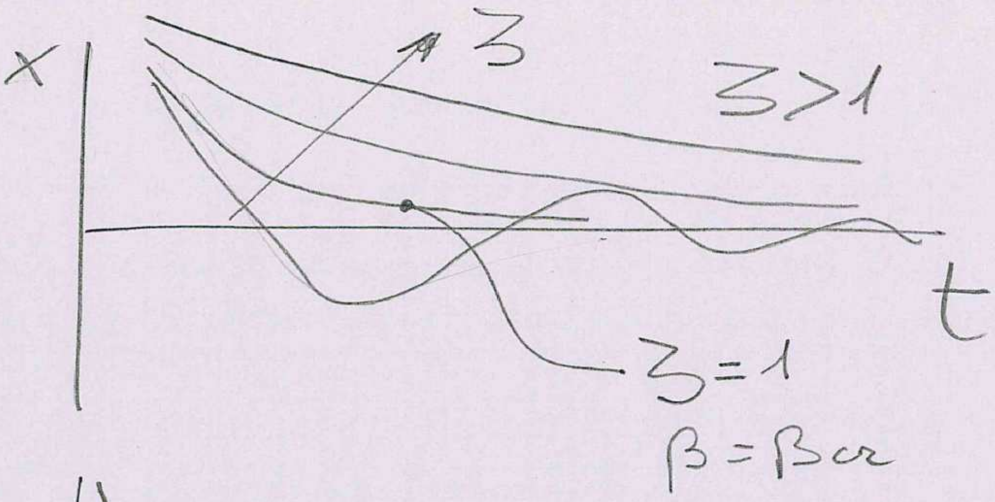
$$\lim_{t\to\infty} x(t) = 0$$

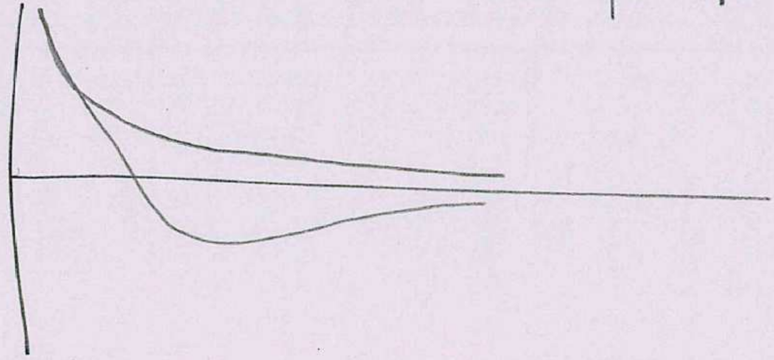
$\zeta = 1$

$$\lambda_1 = \lambda_2 = \lambda = -\omega_n$$

$$x = a e^{\lambda t} + b t e^{\lambda t} = (a + bt) e^{\lambda t}$$

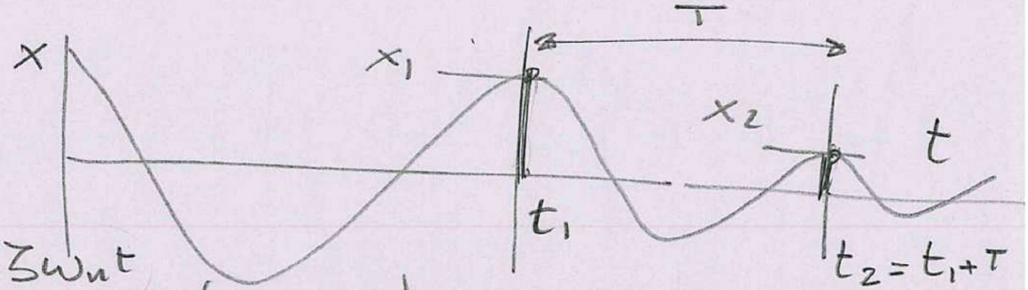






DECREMENTO LOGARITMICO

$\zeta < 1$

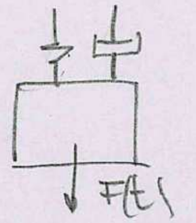


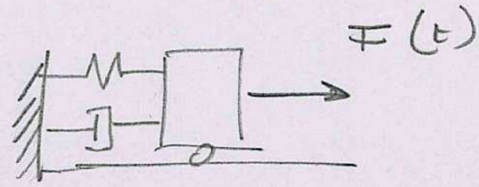
$$x = x_0 e^{-\zeta\omega_n t} \cos(\omega_s t + \varphi_0)$$

$$\frac{x_1}{x_2} = e^{-\zeta\omega_n t_1} \Big/ e^{-\zeta\omega_n(t_1+T)} = e^{\zeta\omega_n T}$$

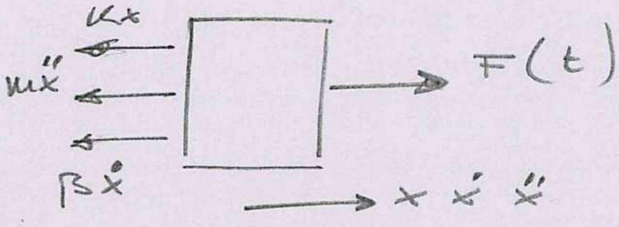
$$\lg \frac{x_1}{x_2} = \zeta\omega_n T = \delta = \zeta\omega_n T = \zeta\omega_n \frac{2\pi}{\omega_s}$$

$$\delta = \zeta\omega_n \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}} \, ; \quad \zeta = \frac{\delta}{\sqrt{4\pi^2+\delta^2}}$$





$$kx_{st} = mg$$



$$m\ddot{x} + \beta\dot{x} + kx = F(t) = F_0 \cos \Omega t$$

$$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = \frac{F_0}{m} \cos \Omega t$$

$$x = x_{g_{OA}} + x_{p_{ec}}$$

$$\lim_{t \to \infty} x = x_{p_{ec}}$$

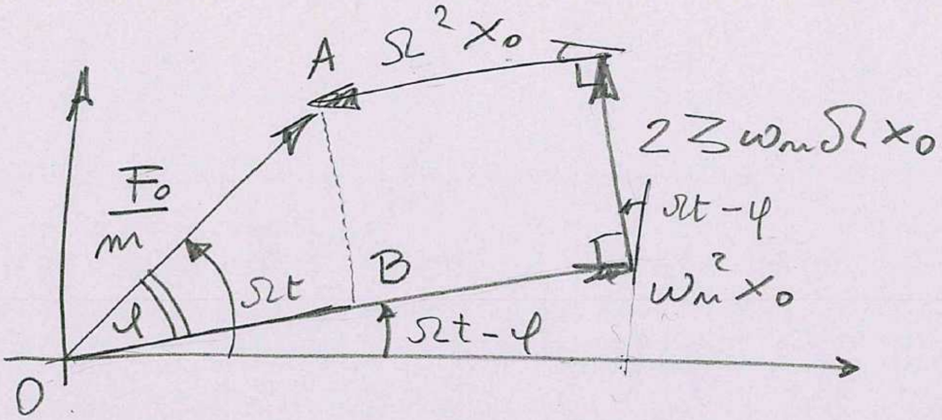
$$x \sim x_p$$
$$\dot{x} \sim \dot{x}_p$$
$$\ddot{x} \sim \ddot{x}_p$$

$$x \sim x_p = x_0 \cos(\Omega t - \varphi)$$

$$\dot{x} = -\Omega x_0 \sin(\Omega t - \varphi)$$

$$\ddot{x} = -\Omega^2 x_0 \cos(\Omega t - \varphi)$$

$$-\Omega^2 x_0 \cos(\Omega t - \varphi) - 2\zeta\omega_n \Omega x_0 \sin(\Omega t - \varphi) +$$
$$+ \omega_n^2 x_0 \cos(\Omega t - \varphi) = \frac{F_0}{m} \cos \Omega t$$



$$OB=(\omega_n^2-\Omega^2)x_0$$

$$AB=2\zeta\omega_n\Omega x_0$$

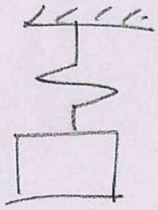
$$OA=\frac{F_0}{m}$$

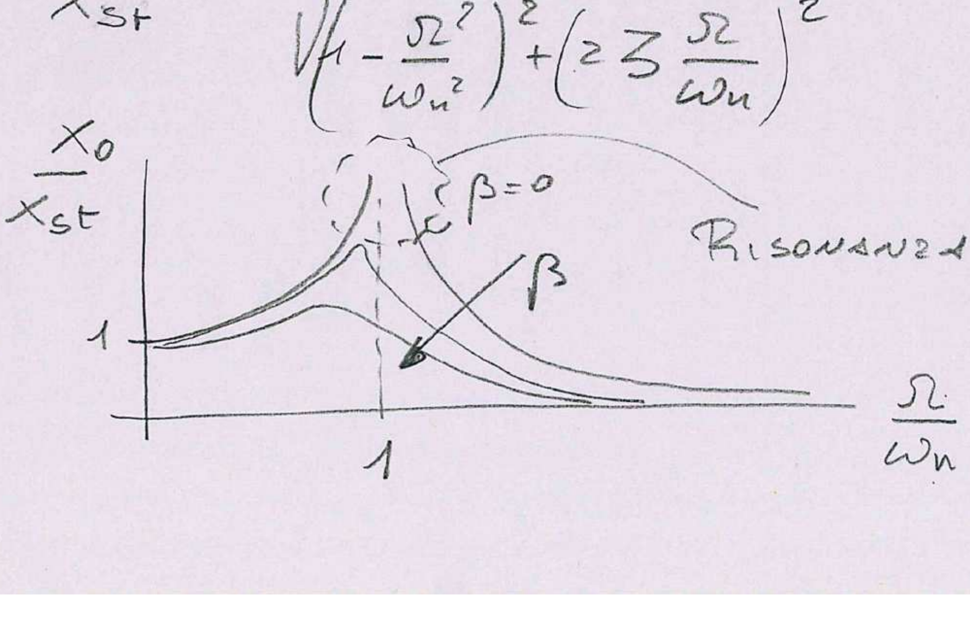
$$OA^2=OB^2+AB^2$$

$$\left(\frac{F_0}{m}\right)^2=\left[(\omega_n^2-\Omega^2)x_0\right]^2+\left[2\zeta\omega_n\Omega x_0\right]^2$$

$$x_0=\frac{F_0/m}{\sqrt{(\omega_n^2-\Omega^2)^2+(2\zeta\omega_n\Omega)^2}}$$

$$\frac{x_0}{x_{st}}=\frac{1}{\sqrt{\left(1-\frac{\Omega^2}{\omega_n^2}\right)^2+\left(2\zeta\frac{\Omega}{\omega_n}\right)^2}}$$



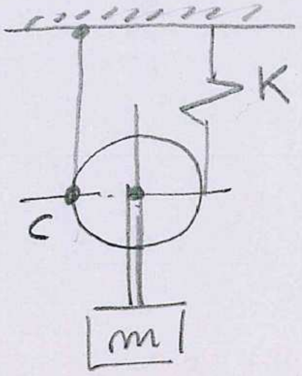


OSCILLAZIONI LIBERA - FORZATA

1 DCL MASSA(E)

2 Eq. EQUILIBRIO

$$\omega_m = \sqrt{K/m}$$



$\overset{\curvearrowleft}{C})$ $m\ddot{x}R + K2x2R = 0$

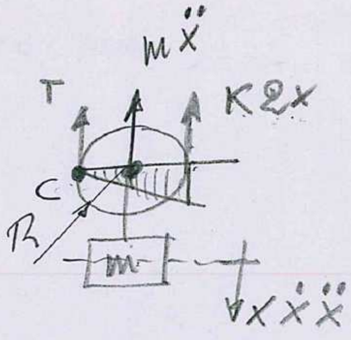
$* \; m\ddot{x} + 4Kx = 0$

$\uparrow)$ $T + m\ddot{x} + K2x = 0$

$TR - K2xR = 0 \quad T = K2x$

$K2x + m\ddot{x} + K2x = 0$

$* \; m\ddot{x} + 4Kx = 0$

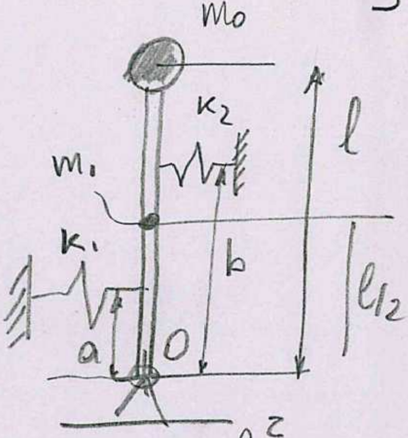


$$\ddot{x} + \frac{4K}{m}x = 0$$

$$\omega_m^2 = \frac{4K}{m} \qquad \omega_n = 2\sqrt{\frac{K}{m}}$$

$$T = \frac{2\pi}{\omega_m}$$

$$m_0 \ddot{\vartheta} l^2 + k_2 \vartheta b^2 + m_1 \ddot{\vartheta} \frac{l^2}{4} + k_1 \vartheta a^2 + I_G \ddot{\vartheta} = 0$$



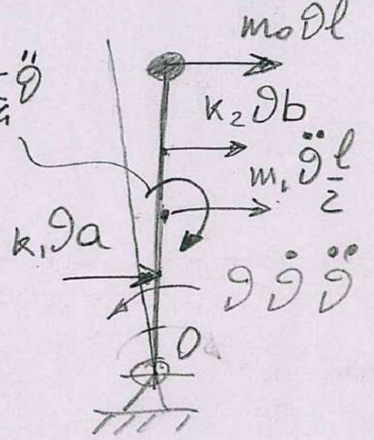
$$\ddot{\vartheta}\left(m_0 l^2 + m_1 \frac{l^2}{4} + I_G\right) + \vartheta\left(k_2 b^2 + k_1 a^2\right) = 0$$

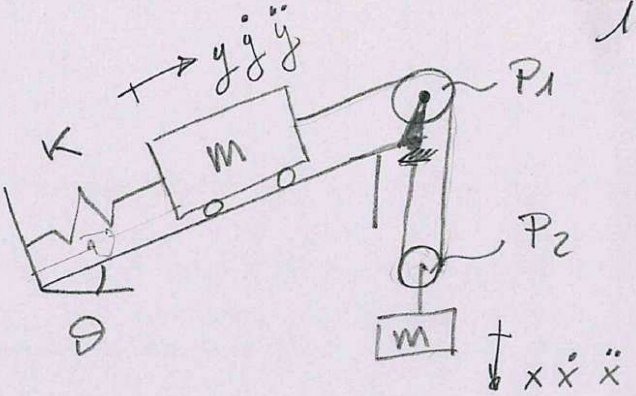
$$\ddot{\vartheta} + \vartheta \frac{k_2 b^2 + k_1 a^2}{m_0 l^2 + m_1 \frac{l^2}{4} + I_G} = 0$$

$$I_G = \frac{m_1 l^2}{12}$$

$$\omega_n = \sqrt{\quad}$$

$$T = \frac{2\pi}{\omega_n}$$





$$2T + m\ddot{x} = 0$$

$$T - ky - m\ddot{y} = 0$$

$$2x = y; \quad 2\ddot{x} = \ddot{y}$$

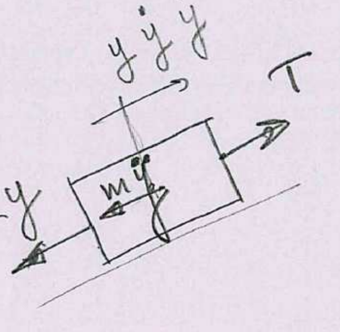
$$2(ky + m\ddot{y}) + m\ddot{x} = 0$$

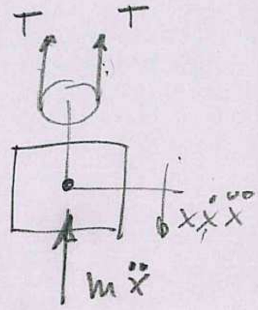
$$2(k\,2x + m2\ddot{x}) + m\ddot{x} = 0$$

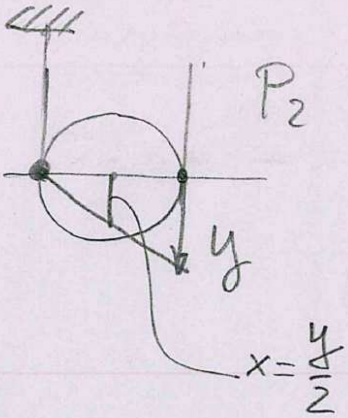
$$\ddot{x}(5m) + x(4k) = 0$$

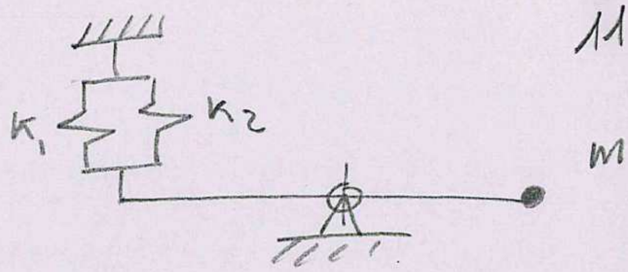
$$\ddot{x} + \frac{4k}{5m}x = 0$$

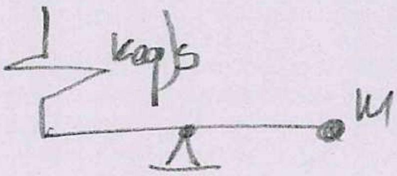
$$\omega_n = \sqrt{\frac{4k}{5m}} \qquad T = \frac{2\pi}{\omega_n}$$

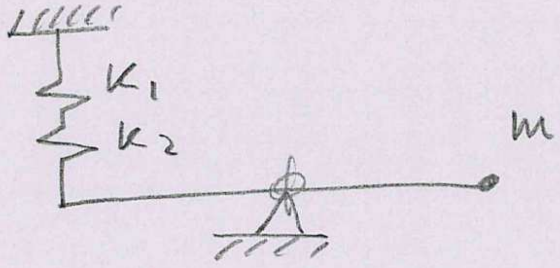


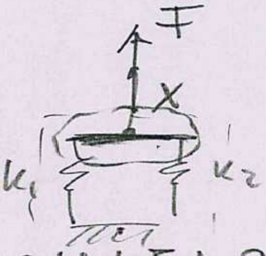


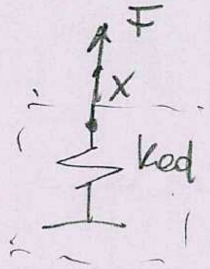


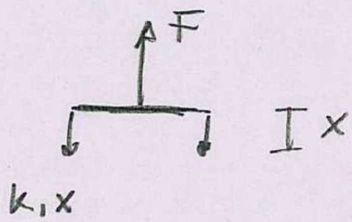




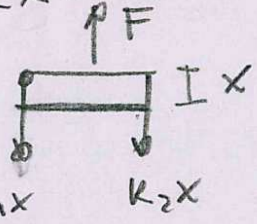






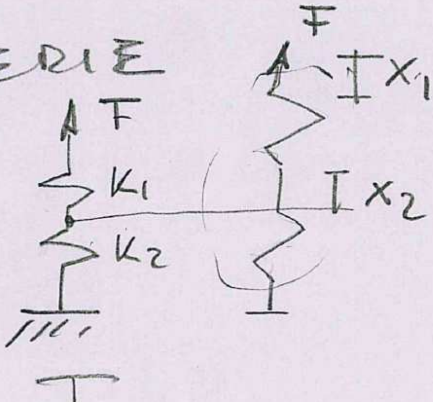


PARALLELO

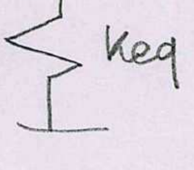


$$F = k_1 x + k_2 x = K_{eq} x$$
$$(k_1 + k_2)x = K_{eq} x$$

SERIE

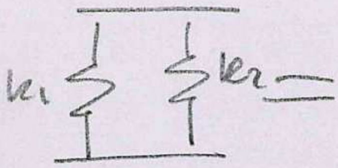


$$F = k_1 x_1 = k_2 x_2$$
$$x_1 = \frac{F}{k_1} \; ; \quad x_2 = \frac{F}{k_2}$$



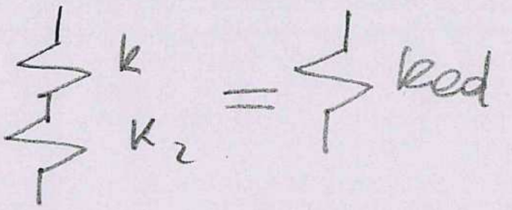
$$F = K_{eq}(x_1 + x_2)$$
$$K_{eq} = \frac{F}{x_1 + x_2} = \frac{F}{\frac{F}{k_1} + \frac{F}{k_2}}$$
$$\frac{1}{K_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

MOLLE IN PARALLELO



$$k_{eq} = k_1 + k_2$$

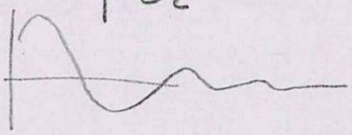
MOLLE IN SERIE

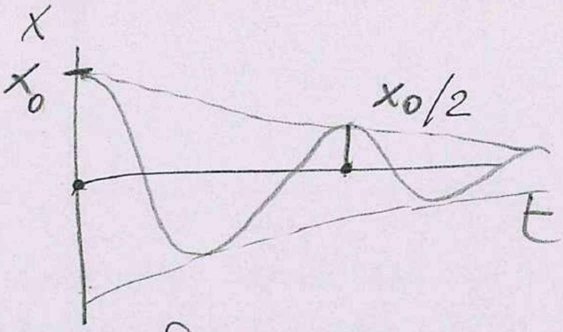


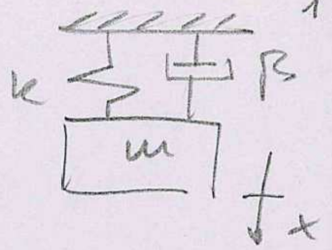
$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$\omega_n = \sqrt{\frac{k}{m}} \qquad \zeta = \frac{\beta}{\beta_{cr}} \qquad \beta_{cr} = 2m\omega_n$$

$$\omega_s = \omega_n\sqrt{1-\zeta^2}$$







$$\zeta = \beta / \beta_{cr}$$

$$x = x_0\, e^{-\zeta \omega_n t} \cos(\omega_s t + \varphi_0)$$

$$\frac{x_1}{x_2} = \frac{e^{-\zeta \omega_n t_1}}{e^{-\zeta \omega_n (t_1 + T)}} \qquad \delta = \lg \frac{x_1}{x_2} = \zeta \omega_n T$$

$$\delta = \lg 2 = 0{,}69 = \zeta \omega_n T = \zeta \omega_n \frac{2\pi}{\omega_s} =$$

$$\omega_s = \omega_n \sqrt{1 - \zeta^2}$$

$$\delta = \zeta \omega_n \frac{2\pi}{\omega_n \sqrt{1 - \zeta^2}} \;; \quad \zeta = \frac{\delta}{\sqrt{\delta^2 + 4\pi^2}} = 0{,}11$$

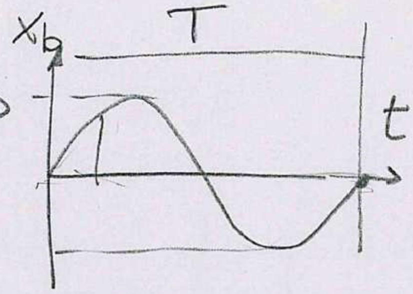
$$\beta_{cr} = 2 m \omega_n$$

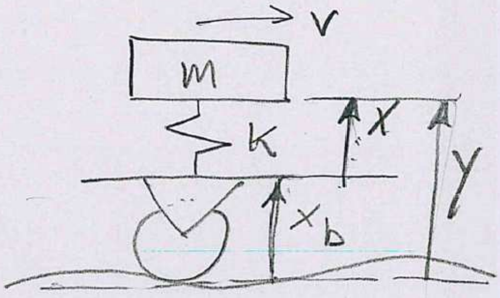
$$\beta = \zeta \beta_{cr}$$

$b = 25\,mm$

$s = 1{,}2\,m$

$V = 25\,km/h$





$$t = \frac{s}{V/3{,}6} = \frac{1{,}2}{25/3{,}6} = 0{,}17\,s$$

$m = 500\,kg$

$\Delta m = 75\,kg \longrightarrow \Delta x = 3\,mm$

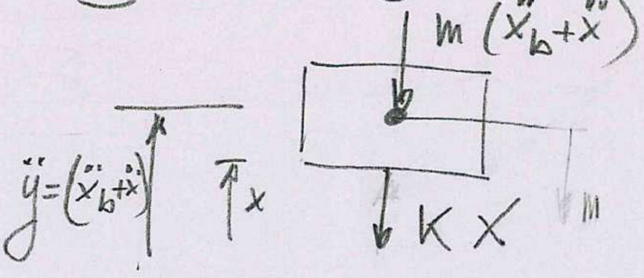
$\Delta F = \Delta m g = 721{,}7\,N$

$\Delta x = 3 \cdot 10^{-3}\,m$

$$k = \frac{\Delta F}{\Delta x} = 240{,}6\,\frac{kN}{m}$$

$$x_b = b \sin \omega t$$

$$T = \frac{s}{V} = \frac{2\pi}{\omega}; \quad \omega = 2\pi \frac{V}{s} = 36{,}34\,\frac{rad}{s}$$



$$m(\ddot{x}_b + \ddot{x}) + kx = 0$$

$$\ddot{x}_b + \ddot{x} + \frac{k}{m} x = 0$$

$$\ddot{x} + \frac{k}{m} x = -\ddot{x}_b \qquad \omega_n = \sqrt{\frac{k}{m}}$$

$$x_b = b \sin \omega t; \quad \dot{x}_b = b\omega \cos \omega t; \quad \ddot{x}_b = -b\omega^2 \sin \omega t$$

$$\ddot{x} + \frac{k}{m} x = b\omega^2 \sin \omega t \qquad x = x_0 \sin \omega t$$

$$\dot{x} = x_0 \omega \cos \omega t$$

$$\ddot{x} = -x_0 \omega^2 \sin \omega t$$

$$-x_0 \omega^2 \sin \omega t + \underbrace{\frac{k}{m}}_{\omega_n^2} x_0 \sin \omega t = b\omega^2 \sin \omega t$$

$$x_0 = b \frac{\omega^2}{\omega_n^2 - \omega^2} = -39{,}75\,mm$$

$$Y_{max} = x_0 + b$$

$$\omega = \omega_M$$

$$\omega = 2\pi \frac{V}{S} = \omega_M$$

$$V = \frac{\omega_M S}{2\pi} = 4{,}2\,m/s \sim 15\,km/h$$

16

$m = 1360\,kg$

$P = 2{,}3\,m$

$x_G = 1{,}3\,m$

$z_G = 0{,}72\,m$

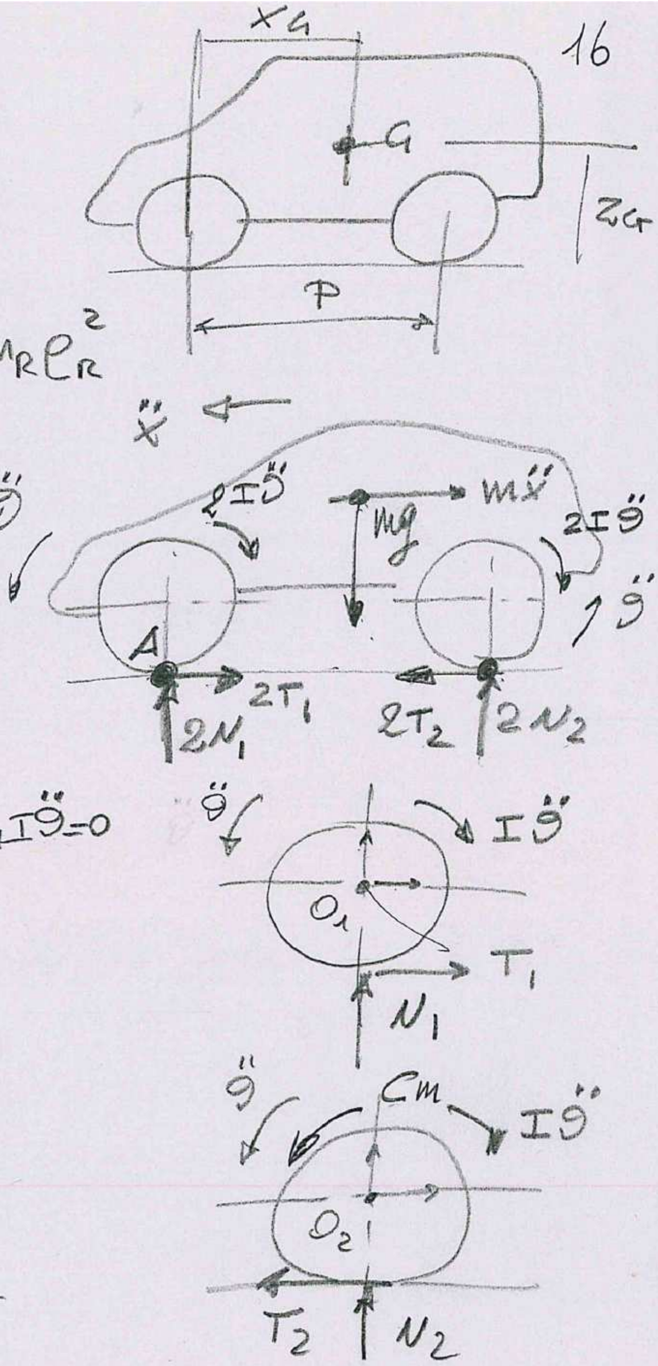
$m_{Ruota} = 10\,kg$

$\rho_{Ruota} = 0{,}2\,m$

$f_a = 1$

$D = 0{,}65\,m$

$I_R = m_R \rho_R^2$



$2T_2 - 2T_1 - m\ddot{x} = 0$

$2N_1 + 2N_2 - mg = 0$

$A)\ 2N_2 P - mg\,x_G - m\ddot{x}z_G - 4I\ddot{\vartheta} = 0$

$T_2 = f_a N_2$

$\ddot{x} = \ddot{\vartheta}\dfrac{D}{2}$

$O_1)\ T_1 \cdot \dfrac{D}{2} - I\ddot{\vartheta} = 0$

$O_2)\ C_m - T_2\dfrac{D}{2} - I\ddot{\vartheta} = 0$

$N_1\ N_2\ T_1\ T_2\ \ddot{x}\ \ddot{\vartheta}\ C_m$

$\ddot{x} = 8{,}02\,m/s^2$