FONDAMENTI DI MECCANICA E BIOMECCANICA [IN/0165]

Lezione del 29 settembre 2017.

Titolo:
Introduzione alla cinematica.

Contenuti:
Definizione di modello fisico, matematico e valutazione di bontà di un modello.
Cinematica del punto.
Sistemi di riferimento ed espressione di posizione, velocità, accelerazione in diversi sistemi di riferimento: cartesiano, polare, locale.
Derivata di grandezza vettoriale.
Cinematica del corpo esteso rigido.
Moti particolari del corpo esteso rigido: moto traslatorio, moto rotatorio. Espressione di posizione, velocità ed accelerazione del corpo e di un suo generico punto.

Riferimento:
Ferraresi C., Raparelli T. "Meccanica applicata - Terza edizione", CLUT, 2007.
Cap. 1 - Elementi di cinematica.
Pagg. 1 - 10

CINEMATICA 29.IX.17 1

MOTO

POSIZIONE (DERIVATE)

TEMPO

MODELLO FISICO

MODELLO MAT.

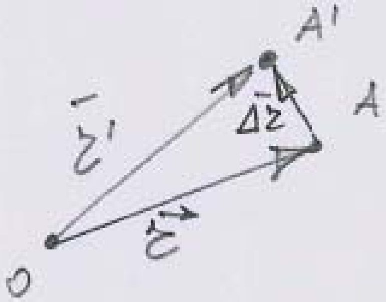
VARIABILI

SPERIMENTAZIONE

CONFRONTO

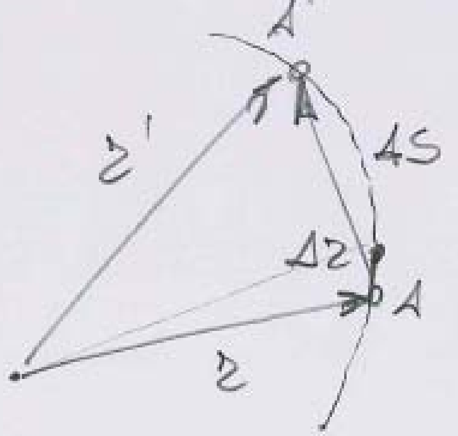
PUNTO MATERIALE

DESCRIZIONE DEL MOTO



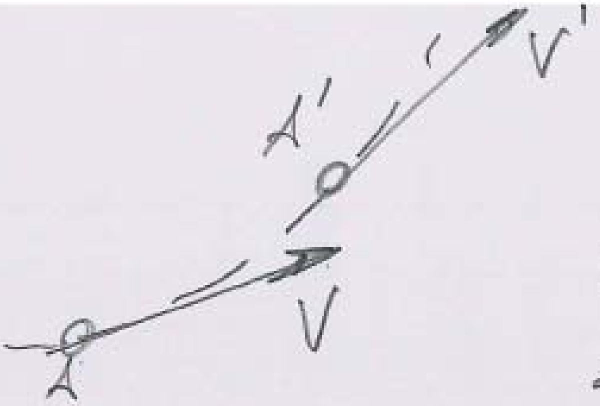
$\bar{r}' = \bar{r} + \bar{\Delta r}$

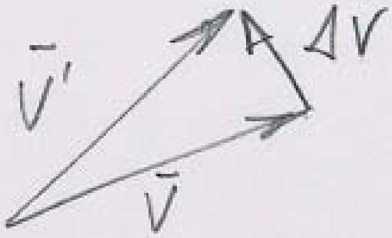
$\Delta S \neq |\Delta \vec{r}|$



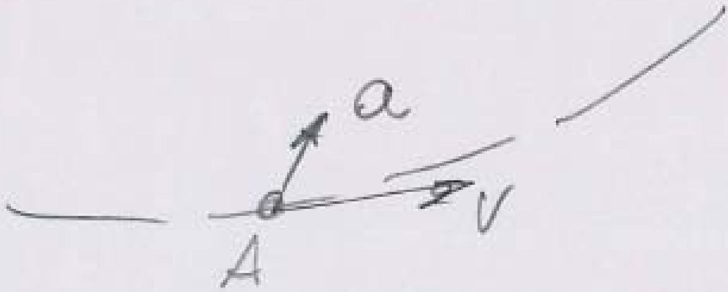
$$\bar{V}_M = \frac{\Delta \vec{r}}{\Delta t}$$

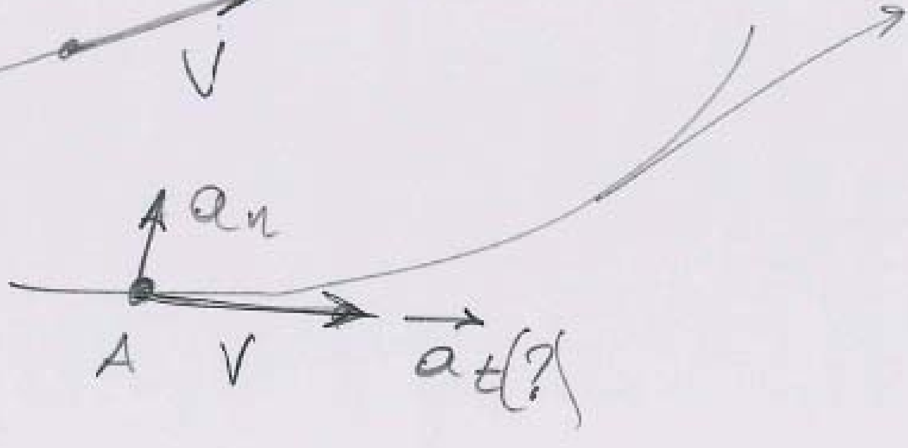
$$\bar{V} = \lim_{\Delta t \to 0} \bar{v}_M = \frac{d\bar{r}}{dt}$$

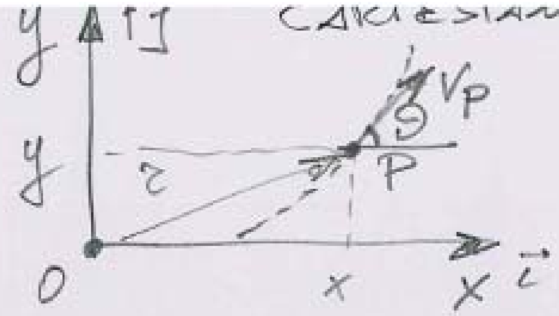




$$\vec{a}_{M} = \frac{\Delta \bar{V}}{\Delta t} \qquad \bar{a} = \lim_{\Delta t \to 0} \bar{a}_{M} = \frac{d\bar{V}}{dt}$$







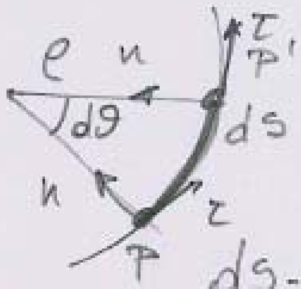
$$\vec{r} = x\vec{i} + y\vec{j}$$

$$\bar{V} = \frac{d\vec{r}}{dt} = \dot{x}\vec{i} + \dot{y}\vec{j} = \bar{v}_x + \bar{v}_y$$

$$\vec{a} = \frac{d\bar{v}}{dt} = \ddot{x}\vec{i} + \ddot{y}\vec{j} = \bar{a}_x + \bar{a}_y$$

$|\bar{V}| = \sqrt{}$ $\quad |a| = \sqrt{}$ $\quad tg\vartheta = v_y/v_x$

LOCALE



$$ds = \rho\, d\vartheta$$

$$\vec{V} = v\vec{\tau}$$

$$v = \frac{ds}{dt} = \rho\frac{d\vartheta}{dt} = \rho\dot{\vartheta};$$

$$\bar{V} = \rho\dot{\vartheta}\vec{\tau}$$

$$\vec{a} = \frac{d\bar{V}}{dt} = \frac{d(v\bar{\tau})}{dt} = \dot{v}\bar{\tau} + v\dot{\bar{\tau}} =$$

$$= \dot{v}\bar{\tau} + v\bar{\omega}\wedge\bar{\tau} =$$

$$= \dot{v}\vec{\tau} + v\dot{\vartheta}\bar{k}\wedge\bar{\tau} =$$

$$= \dot{v}\bar{\tau} + v\dot{\vartheta}\bar{n}$$

$$\vartheta = \dot{\vartheta}\Delta t \qquad \bar{\omega} = \dot{\vartheta}\bar{k}$$

$$\Delta\bar{\lambda} = |\lambda|\,\vartheta\,\bar{\mu}$$

$$\Delta\bar{\lambda} = |\lambda|\,\dot{\vartheta}\Delta t\,\bar{\mu}$$

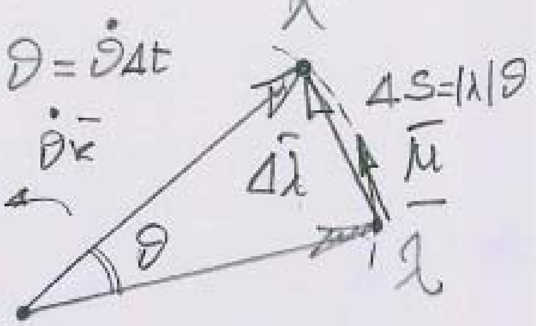
$$\frac{\Delta\bar{\lambda}}{\Delta t} = 1\cdot\dot{\vartheta}\bar{\mu} = \dot{\vartheta}\bar{\mu}$$

$$\frac{d\bar{\lambda}}{dt} = \dot{\vartheta}\bar{\mu} = \underline{\underline{\bar{\omega}\wedge\bar{\lambda}}} =$$

$$= \dot{\vartheta}\bar{k}\wedge\bar{\lambda}$$

DERIVATA DI UN VERSORE

$$\frac{d\bar{\lambda}}{dt} \; ?$$



$$\Delta\bar{\lambda} = |\lambda| \vartheta \vec{\mu}$$

$$\Delta\bar{\lambda} = |\lambda| \dot{\vartheta} \Delta t \bar{\mu}$$

$$\frac{\Delta\bar{\lambda}}{\Delta t} = \dot{\vartheta}\vec{\mu}$$

$$\bar{\omega} = \dot{\vartheta} k$$

$$\frac{d\bar{\lambda}}{dt} = \bar{\omega} \wedge \bar{\lambda}$$

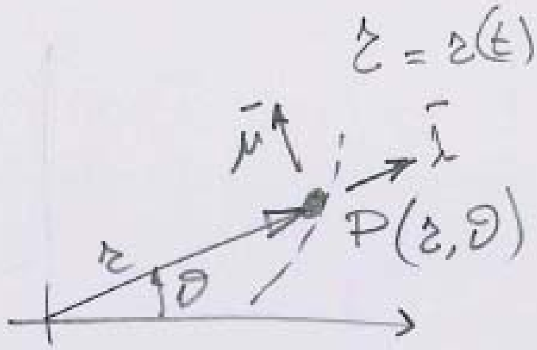
$$\bar{\omega} = \dot{\vartheta}\bar{k}$$

$$\bar{\omega} \wedge \bar{\lambda} = \dot{\vartheta}\bar{k} \wedge \bar{\lambda} = \dot{\vartheta}\bar{\mu}$$

$$= \bar{\omega} \wedge \bar{\lambda}$$

POLARE

$$r = r(t) \qquad \vartheta = \vartheta(t)$$



$$\bar r = r\bar\lambda$$

$$\dot{\bar\lambda} = \frac{d\bar\lambda}{dt} = \bar\omega\wedge\bar\lambda = \dot\vartheta\bar\mu$$

$$\dot{\bar\mu} = \frac{d\bar\mu}{dt} = \bar\omega\wedge\bar\mu = -\dot\vartheta\bar\lambda$$

$$\bar V = \frac{d\bar r}{dt} = \dot r\bar\lambda + r\dot{\bar\lambda}$$

$$\bar V = \dot r\bar\lambda + r\dot\vartheta\bar\mu$$

$$\bar a = \frac{d\bar V}{dt} = \ddot r\bar\lambda + \dot r\dot{\bar\lambda} + \dot r\dot\vartheta\bar\mu + r\ddot\vartheta\bar\mu + r\dot\vartheta\dot{\bar\mu}$$

$$a = \ddot r\bar\lambda + \dot r\dot\vartheta\bar\mu + \dot r\dot\vartheta\bar\mu + r\ddot\vartheta\bar\mu + r\dot\vartheta^2(-\bar\lambda)$$

$$a = \bar\lambda\left(\ddot r - r\dot\vartheta^2\right) + \bar\mu\left(2\dot r\dot\vartheta + r\ddot\vartheta\right)$$

$$a = a_\lambda\bar\lambda + a_\mu\bar\mu$$

RADIALE TRASVERSALE

- MOTO RETTILINEO UNIFORME

$\longrightarrow x$

$$\bar{V} = \dot{\bar{x}} = \overline{\text{cost}} \qquad \bar{a} = \frac{d\bar{V}}{dt} = \bar{0}$$

$$x(t) = \int_{t_0}^{t} V dt + x_0$$

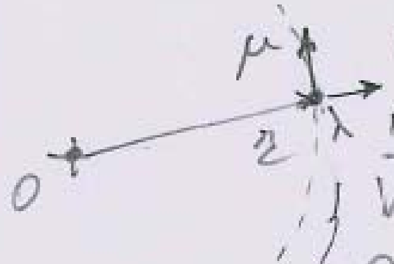
- MOTO UNIF. ACC. RETTILINEO

$$\bar{a} = \dot{\bar{V}} = \ddot{\bar{x}} = \overline{\text{cost}}$$

$$V(t) = \int_{t_0}^{t} a\, dt + V_0 = a(t - t_0) + V_0$$

$$x(t) = \int_{t_0}^{t} V dt + x_0 = \frac{1}{2} a (t - t_0)^2 + V_0 (t - t_0) + x_0$$

- MOTO CIRCOLARE



$$r = \text{cost} \qquad \dot{r} = 0 \qquad \ddot{r} = 0$$

IN GENERALE (COORD. POLARI)

$$\begin{cases} \bar{V} = \dot{r}\bar{\lambda} + r\dot{\vartheta}\bar{\mu} \\ a = (\ddot{r} - r\dot{\vartheta}^2)\bar{\lambda} + (2\dot{r}\dot{\vartheta} + r\ddot{\vartheta})\bar{\mu} \end{cases}$$

$$\bar{V} = r\dot{\vartheta}\bar{\mu}$$

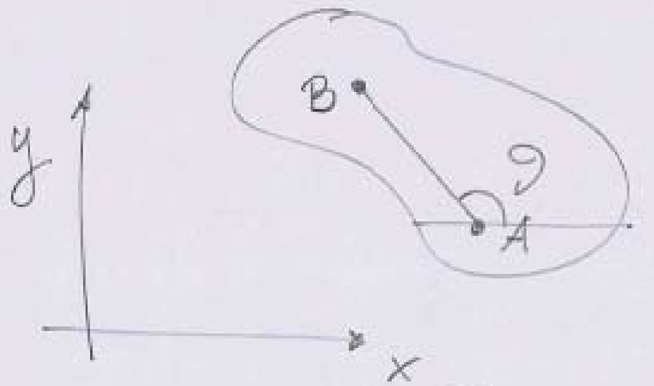
$$\bar{a} = -r\dot{\vartheta}^2\bar{\lambda} + r\ddot{\vartheta}\bar{\mu}$$

CINEMATICA

CORPO ESTESA. RIGIDO

C. RIGIDO: CORPO I PUNTI NON CAMBIANO LA MUTUA POSIZIONE

MOTO TRASLATORIO
ROTATORIO
PIANO GENERICO

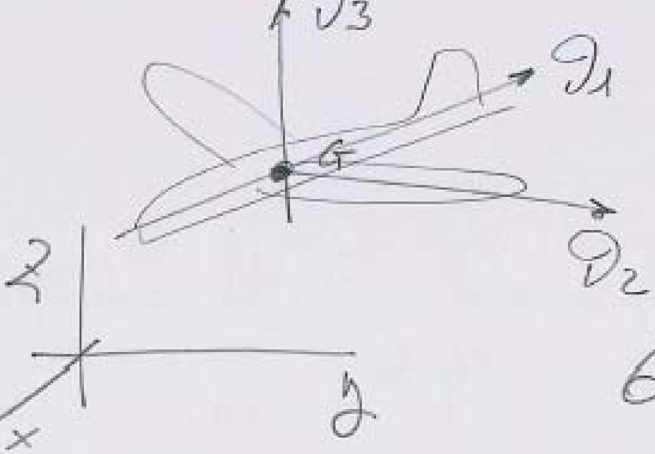


2D

A

x_A, y_A, ϑ

3 GRADI DI LIBERTA'

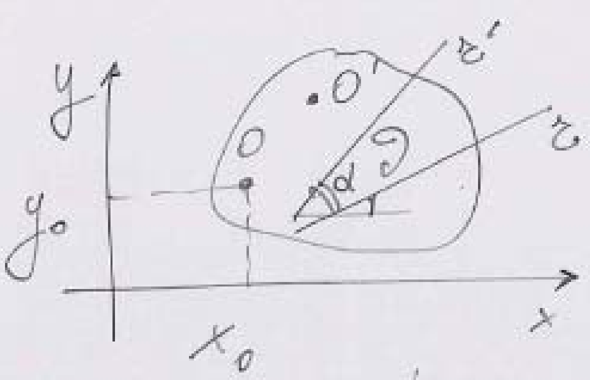


3D

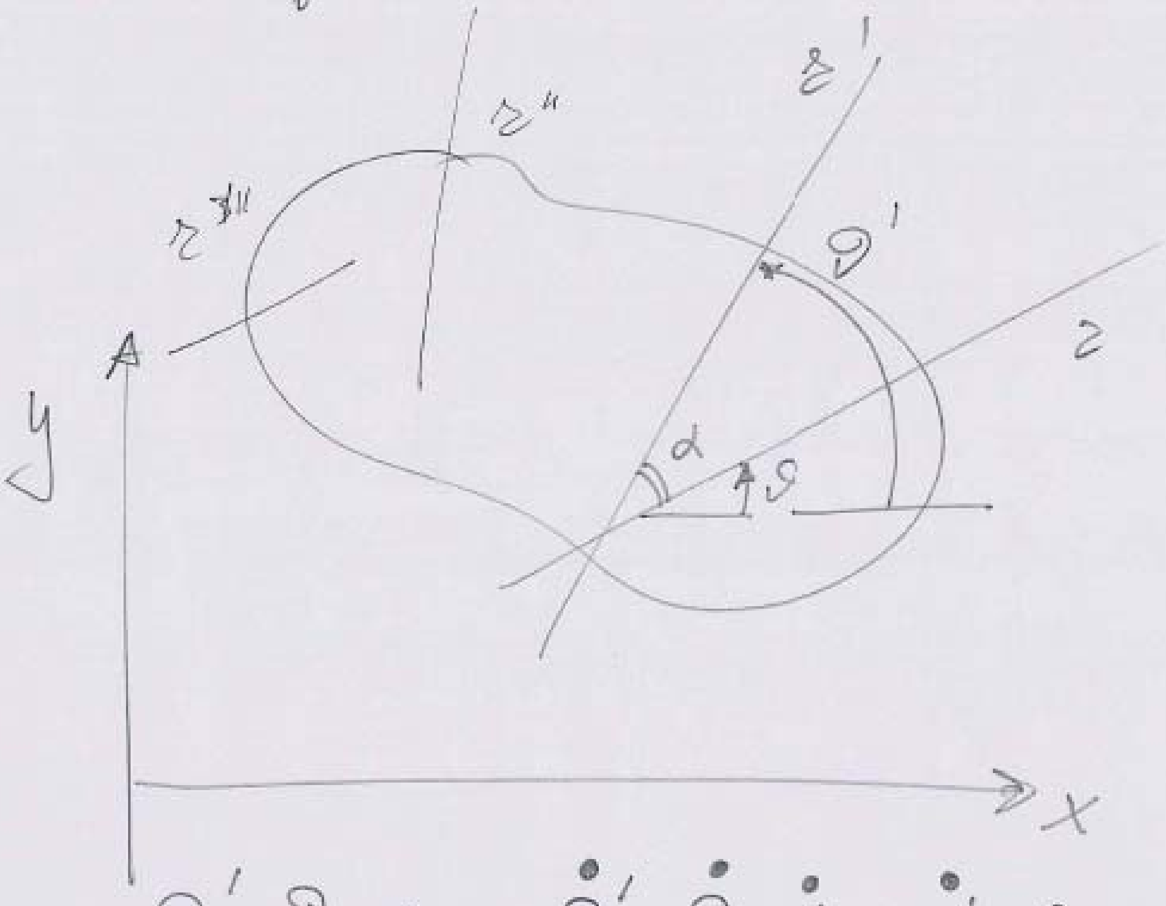
$x_G\ y_G\ z_G$

$\vartheta_1\ \vartheta_2\ \vartheta_3$

6 GRADI DI LIBERTA'



x_0, y_0, ϑ

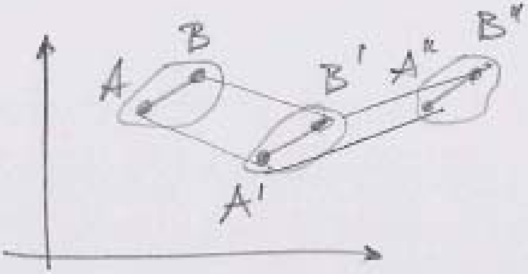


$\vartheta' = \vartheta + \alpha \qquad \dot{\vartheta}' = \dot{\vartheta} + \dot{\alpha} \qquad \dot{\alpha} = 0$

$\ddot{\vartheta}' = \ddot{\vartheta}$

VEL. E ACC. ANG. $\Rightarrow$ COMUNI INTERO CORPO RIGIDO

MOTO TRASLATORIO



SEMPRE PARALLELO A SÉ STESSO

$D = \text{COST} \quad \omega = 0$

$\overline{AA'} = \overline{BB'}$

$$\vec{V}_A = \lim_{\Delta t \to 0} \frac{\overline{AA'}}{\Delta t}$$

$$\bar{V}_B = \lim_{\Delta t \to 0} \frac{\overline{BB'}}{\Delta t}$$

$AA' = BB'$

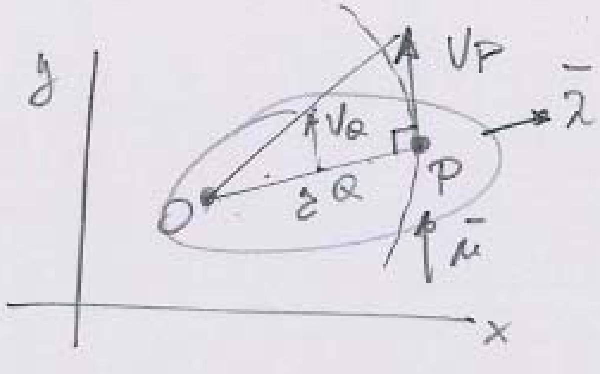
$$\bar{V}_A = \bar{V}_B$$

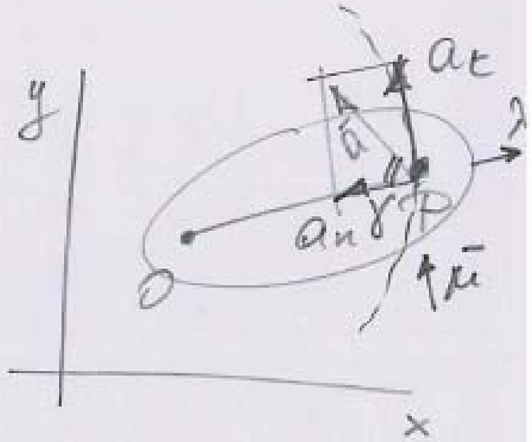
$$\bar{a}_A = \bar{a}_B$$

TUTTI I PUNTI DEL CORPO STESSA VELOCITÀ

MOTO ROTATORIO

PUNTO FISSO



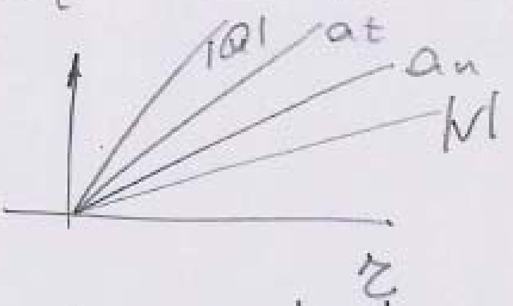


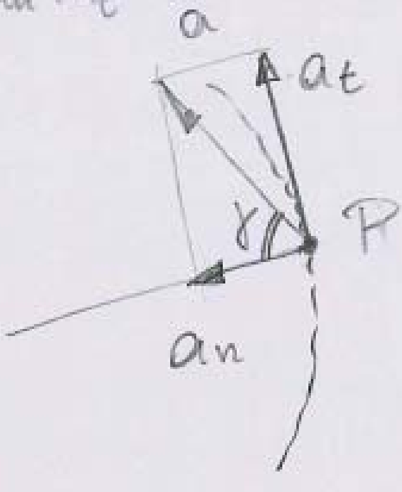
$\bar{V}_P = \omega r \bar{\mu}$; $|V_P| = \omega r$

$\bar{a}_n = - r\omega^2 \lambda$

$\bar{a}_t = r \dot{\omega} \mu$

$|a_P| = \sqrt{a_n^2 + a_t^2}$





$$tg\gamma = \frac{|a_t|}{|a_n|}$$

$\gamma = 0 \quad |a_t| = r\dot{\omega} = 0 \Rightarrow \dot{\omega} = 0 \Rightarrow \omega = \text{COST}$

$\gamma = \frac{\pi}{2}\,\text{rad} \quad |a_n| = r\omega^2 = 0 \Rightarrow \omega = 0, \ \dot{\omega} \neq 0$ MOTO INCIPIENTE