FONDAMENTI DI MECCANICA E BIOMECCANICA [IN/0165]

Lezione del 27 ottobre 2017.

Titolo:
Potenza, Rendimento, Quantità di moto.

Contenuti:
Definizione e unità di misura di Potenza, Rendimento, Quantità di moto.
Azioni motrici e resistenti in un sistema, verso di azioni (forze e momenti) e velocità (lineari ed angolari.
Quantità di moto e momento della quantità di moto. Conservazione della quantità di moto e del momento della quantità di moto.
Esempi di sistemi e verso di azioni e velocità motrici e resistenti.
Urto di carrelli.

Riferimento:
Ferraresi C., Raparelli T. "Meccanica applicata - Terza edizione", CLUT, 2007.
Cap. 1 – Dinamica – Lavoro ed energia.
Pagg. 65 – 92
Legnani G., Palmieri G. "Fondamenti di meccanica e biomeccanica del movimento", CittàStudi, 2016.
Cap. 3 – Dinamica.
Pagg. 255 – 303

27.x.17

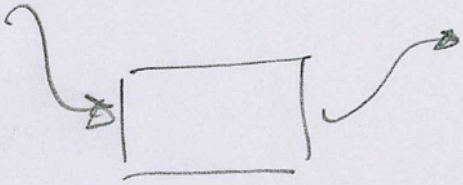
POTENZA

$$P = \frac{dL}{dt} \qquad \frac{J}{s} = W$$

$$\frac{kg\ \frac{m}{s^2}\ m}{s}$$

RENDIMENTO

$$\eta = \frac{\text{UTILE}}{\text{SPESA}} = \frac{\text{Pot UTILE}}{\text{Pot SPESA}}$$



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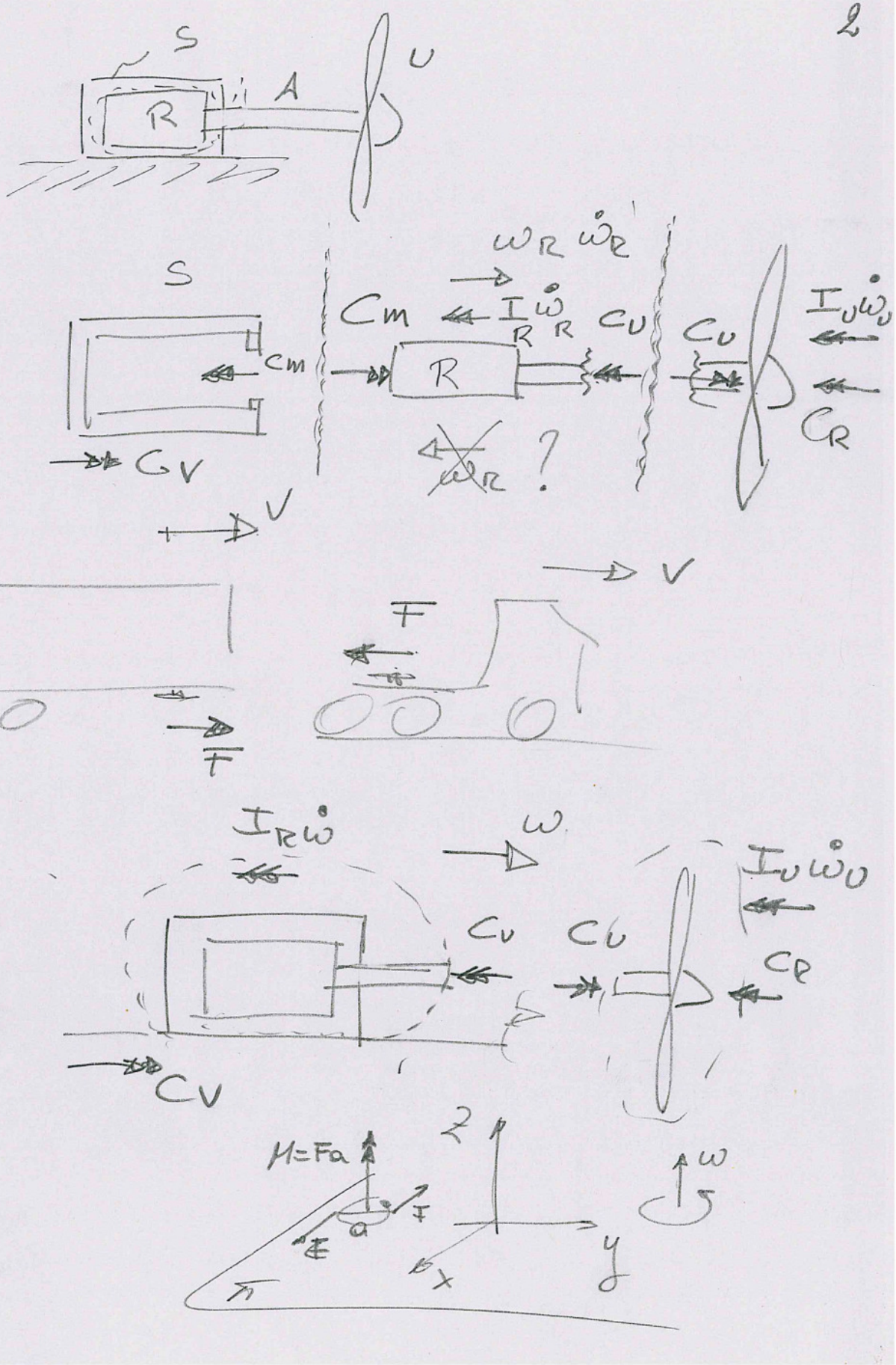
S
A
R
U
S
Cm
cm
CV
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ωR ω̇R
Cm
I ω̇ R
R
CU
CU
I U ω̇U
CR
ωR ?
V
F
F
I R ω̇
ω
CU
CU
I U ω̇U
CR
CV
M=Fa
a
F
E
x
z
y
ω

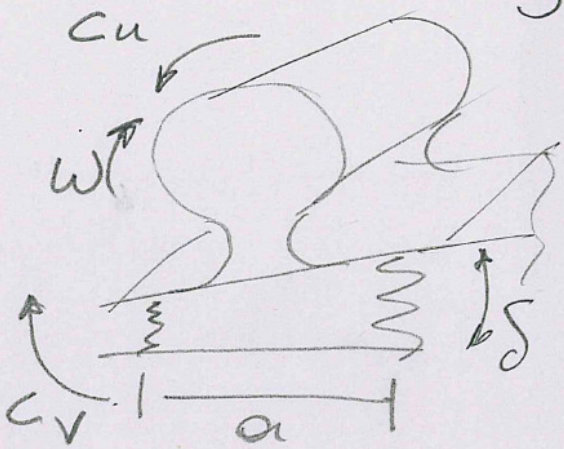
$a = 250\ mm$

$P = 5\ kW$

$n = 1500\ g/1'$

$K = 15\ kN/m$

δ ?

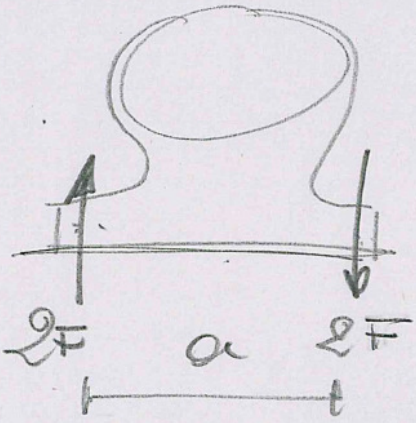


$C_v = 2F \cdot a$

$F = kx$

$\delta = \operatorname{atan} \frac{2x}{a}$

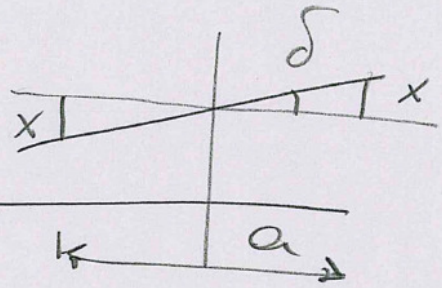
$C_v - C_u = 0$



$Pot = C \cdot \omega$

$L = F \cdot S \qquad P = \frac{L}{t} = F\frac{S}{t} = FV$

$L = M \cdot \vartheta \qquad P = \frac{L}{t} = M\frac{\vartheta}{t} = M \cdot \omega$



$Pot = C \cdot \omega$

$C_u = \frac{Pot}{\omega} = \frac{5 \cdot 10^3}{157,08}$

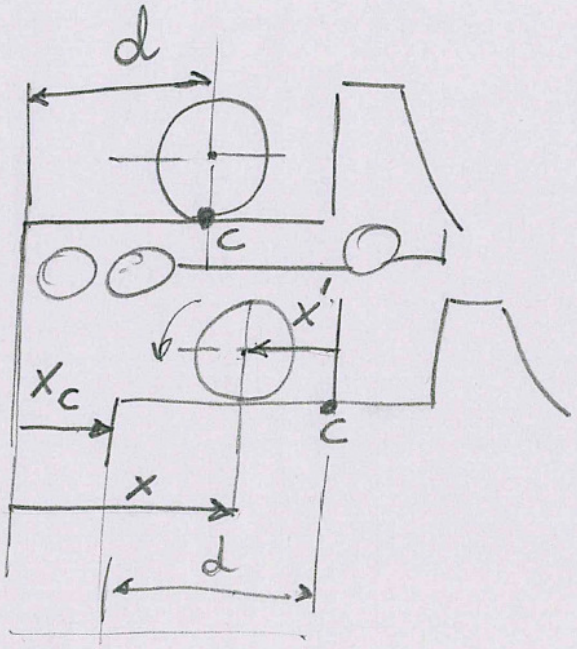
$\omega = \frac{1500 \cdot 2\pi}{60} = 157,08\ \frac{rad}{s}$

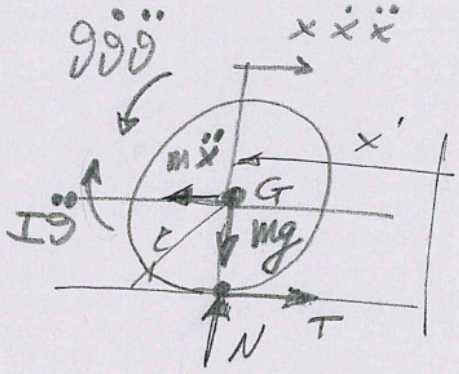
$C_u = 31,83\ N\,m$

$F = \frac{C}{2a} = 63,66\ N; \quad x = \frac{F}{K} = \frac{63,66}{15 \cdot 10^3} = 4,24 \cdot 10^{-3}\ m$

$\delta = \operatorname{atan} 2x/a = 0,03\ rad$

$$I = \frac{m r^2}{2}$$





$$x' = \vartheta r$$

$$\vartheta = \frac{x'}{r};\quad \ddot{\vartheta} = \frac{\ddot{x}'}{r}$$

$$\begin{cases} T - m\ddot{x} = 0 \\ N - mg = 0 \\ Tr - I\ddot{\vartheta} = 0 \end{cases}$$

$$x_c = \frac{1}{2}\ddot{x}_c t^2$$

$$x + x' = x_c + d$$

$$\ddot{x} + \ddot{x}' = \ddot{x}_c$$

$$x_c = \frac{1}{2}(\ddot{x} + \ddot{x}')t^2$$

$$x_c = \frac{1}{2}\left(\frac{\ddot{x}'}{2} + \ddot{x}'\right)t^2$$

$$x_c = \frac{1}{2}\frac{3}{2}\ddot{x}' t^2 = \frac{3}{4}\ddot{x}' t^2$$

$$d = \frac{1}{2}\ddot{x}' t^2;\quad t^2 = \frac{2d}{\ddot{x}'}$$

$$x_c = \frac{3}{4}\ddot{x}'\frac{2d}{\ddot{x}'} = \frac{3}{2}d$$

$$\ddot{x} = \frac{\ddot{x}'}{2}$$

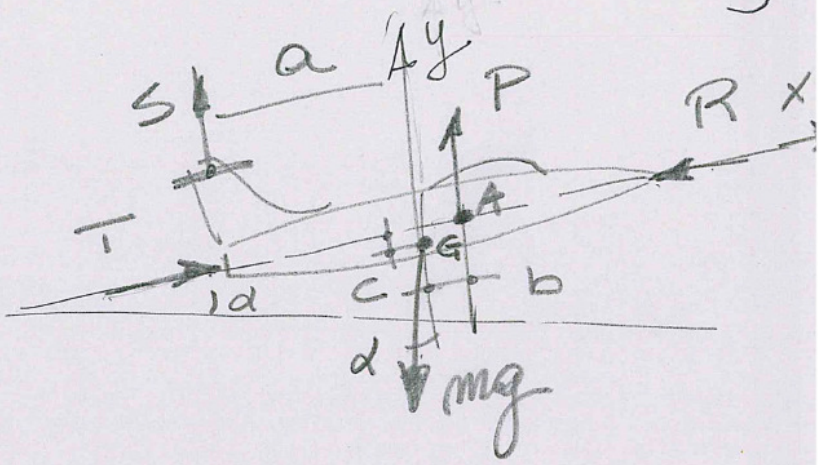
$$T = m\ddot{x}$$

$$Tr = \frac{mr^2}{2}\frac{\ddot{x}'}{r} = \frac{mr}{2}\ddot{x}'$$

$$T = m\frac{\ddot{x}'}{2}$$

$$m\ddot{x} = m\frac{\ddot{x}'}{2}$$

$\alpha = 15°$
$m = 30.10^3 kg$
$R = 10.10^3 N$
$a = 9 m$
$b = 200 mm$
$c = 180 mm$
T?



x) $T - R - mg \sin\alpha = 0$

y) $P + S - mg \cos\alpha = 0$

A) $b.mg.\cos\alpha - c.mg.\sin\alpha - S.(a+b) = 0$

$$S = \frac{1}{a+b} mg(b\cos\alpha - c\sin\alpha)$$

$$S = \frac{1}{9+0,2} 30.10^3.9,81(0,2.\cos 15° - 0,180.\sin 15°)$$

$$S = \frac{1}{9,2} \cdot 294,3.10^3 (0,147) = 4689,53 N$$

$P = mg\cos\alpha - S = 30.10^3.9,81.\cos 15 - 4689,53$

$P = 279582,44 N$

$T = R + mg\sin\alpha = 10.10^3 + 30.10^3.9,81.\sin 15°$

$T = 86,120 kN$

$\vec{a} = \frac{\bar{R}}{m}$ QUANTO DI MOTO

$\bar{R} - m\bar{a} = \bar{0}$ $\bar{Q} = m\bar{V}$

F_i

$\bar{R} = m\bar{a} = m\frac{d\bar{V}}{dt} = \frac{dm\bar{v}}{dt}$

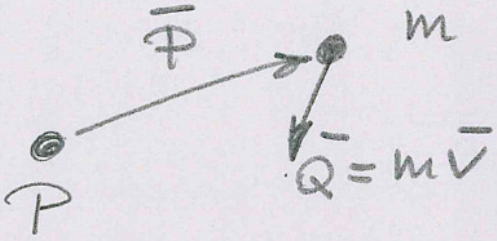
$\bar{R} = \frac{d}{dt}\bar{Q}$

$\frac{d\bar{Q}}{dt} = \bar{0}$ SE $\bar{R} = \bar{0}$

$\bar{Q} = COST$ SE $\bar{R} = \bar{0}$

$$\bar{Q} = m\bar{v}$$

$$\bar{K}_P = \bar{P} \wedge m\bar{v}$$



$$\frac{d\bar{K}_P}{dt} = \frac{d\bar{P}}{dt} \wedge m\bar{v} + \bar{P} \wedge m\frac{d\bar{v}}{dt}$$

$$\bar{v} \wedge m\bar{v} + \bar{P} \wedge m\bar{a}$$

$\bar{v} \wedge m\bar{v} = 0$

$$\bar{P} \wedge \bar{R}$$

$$\bar{M}_P$$

$$\bar{R} = \frac{d\bar{Q}}{dt}$$

$$\bar{M} = \frac{d\bar{K}}{dt}$$

1 EQ. EQUILIBRIO

$$\sum \bar{F} = \bar{0}$$

$$\sum \bar{M} = \bar{0}$$

2 BILANCIO ENERGIA

$$L_e + L_i = \Delta E_c + \Delta E_g + \Delta E_k + \dots$$

3

$$\bar{R} = \frac{d\bar{Q}}{dt} \qquad Q = mV$$

$$R = 0 \qquad Q = \text{cost}$$

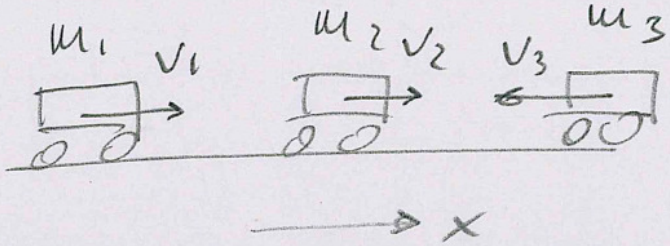
$$\bar{M} = \frac{d\bar{K}}{dt} \qquad \bar{K} = \bar{P} \wedge \bar{Q}$$

$$M = 0 \qquad K = \text{cost}$$

$V_1 = 2\ km/h$

$V_2 = 1\ km/h$

$V_3 = 1,5\ km/h$



$m_1 = 65 \cdot 10^3\ kg \quad m_2 = 50 \cdot 10^3\ kg \quad m_3 = 75 \cdot 10^3\ kg$

$\Delta \vec{Q} = \vec{0}$ PRIMA - DOPO URTO

PRIMA

$$\begin{cases} Q_I = m_1 V_1 + m_2 V_2 - m_3 V_3 \\ Q_I = 65.10^3 \cdot 0,55 + 50.10^3 \cdot 0,28 - 75.10^3 \cdot 0,42 \\ Q_I = 18,5.10^3\ kg\ \frac{m}{s} \end{cases}$$

$$Q_{II} = (m_1 + m_2 + m_3)\, V_{II} = Q_{II} \Big/ \vec{R} = 0$$

$$V_{II} = \frac{Q_I}{m_1 + m_2 + m_3}$$

$$V_{II} = \frac{18,5.10^3}{190 \cdot 10^3} = 0,1\ m/s$$

L_i PERDUTO $= \Delta E_c =$

$$= \frac{1}{2}(m_1 + m_2 + m_3) V_{II}^2 - \left[\frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 + \frac{1}{2} m_3 V_3^2\right]$$