FONDAMENTI DI MECCANICA E BIOMECCANICA [IN/0165]

Lezione del 24 novembre 2017.

Titolo:
Applicazioni delle leggi della dinamica con attrito.
Oscillazioni libere ad un grado di libertà.

Contenuti:
Applicazioni delle leggi della dinamica con attrito.
Sistemi di sollevamento con carrucole, in presenza di attrito al perno. Puleggia fissa e puleggia mobile.
Analisi dinamica del moto di una bicicletta in partenza con accelerazione massima: analisi delle forze scambiate con la pista, accelerazione massima, forza massima sulla pedivella.
Oscillazioni libere.
analisi di un sistema oscillante in moto libero. Sistema non smorzato e smorzato.

Riferimento:
Ferraresi C., Raparelli T. "Meccanica applicata - Terza edizione", CLUT, 2007.
Cap. 3 – Attrito.
Pagg. 93 - 104
Cap. 7 – Vibrazioni lineari ad un grado di libertà.
Pagg. 203 - 238
Legnani G., Palmieri G. "Fondamenti di meccanica e biomeccanica del movimento", CittàStudi, 2016.
Cap. 4.4 – Meccanica delle vibrazioni.
Pagg. 329 - 362

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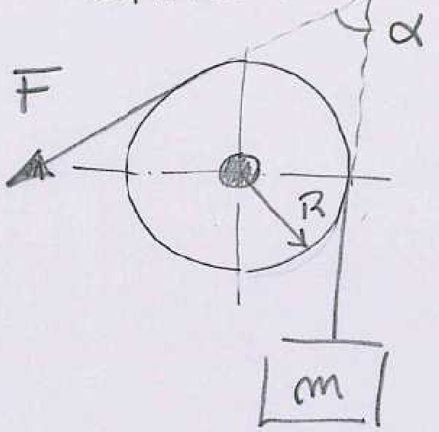
R . R. PUL.

r R. PERNO

φ ATTRITO

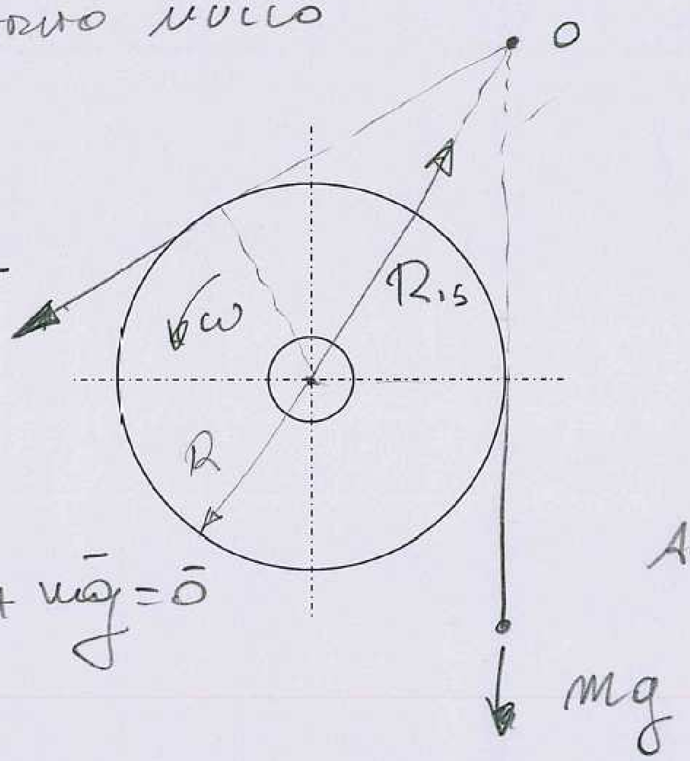
α

m



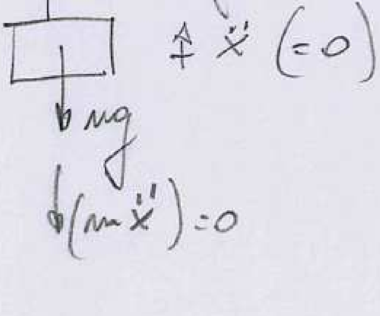
ATTRITO NULLO

DCL PULEGGIA



ACC. = COST

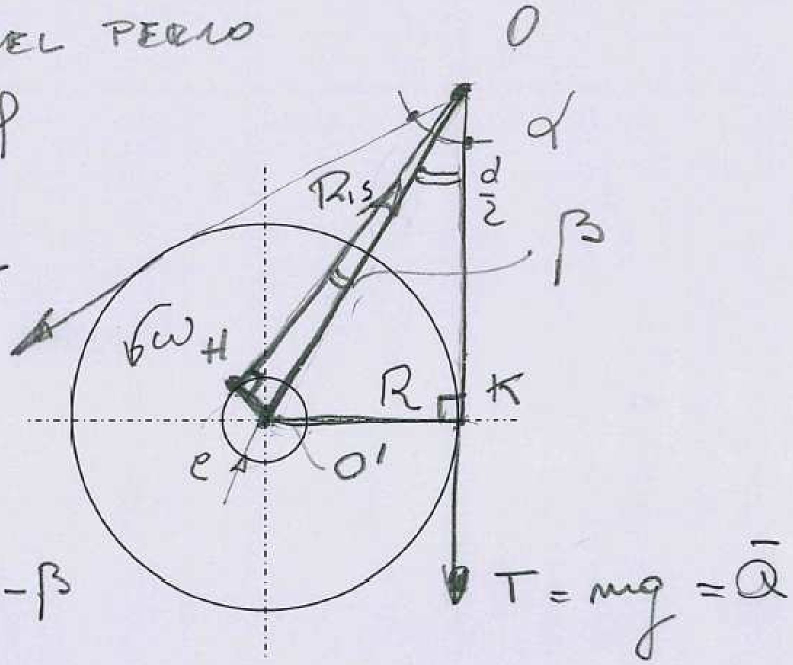
$$\bar{R}_{15} + \vec{F} + m\bar{g} = \bar{0}$$

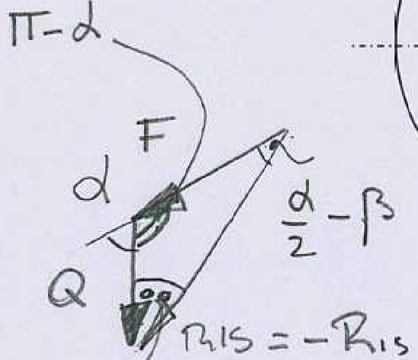


ATTRITO NEL PERNO

$\rho = r \sin\varphi$

$\vec{F} + \bar{Q} = \bar{R}$





$$\gamma = \pi - \left[(\pi - \alpha) + \left(\frac{\alpha}{2} - \beta\right)\right] = \frac{\alpha}{2} + \beta$$

$$\frac{F}{\sin\gamma} = \frac{Q}{\sin\left(\frac{\alpha}{2} - \beta\right)}$$

$\rho = r \sin\varphi$ | (*) $\rho = OO' \sin\beta$ $\left(\overset{\triangle}{OO'H}\right)$

(*) $O'O = \dfrac{R}{\sin\frac{\alpha}{2}}$ $\left(\overset{\triangle}{O'KO}\right)$

(*) $\sin\beta = \dfrac{\rho}{R} \sin\dfrac{\alpha}{2}$

$\beta = \arcsin\left(\frac{\rho}{R} \sin\frac{\alpha}{2}\right)$; $F = Q \dfrac{\sin\gamma}{\sin\left(\frac{\alpha}{2} - \beta\right)}$

NO ATTRITO

1) $T R_1 - T_1 R_1 = 0$

$T_1 = T$

2) $T_1 R_2 - T_2 R_2 = 0$

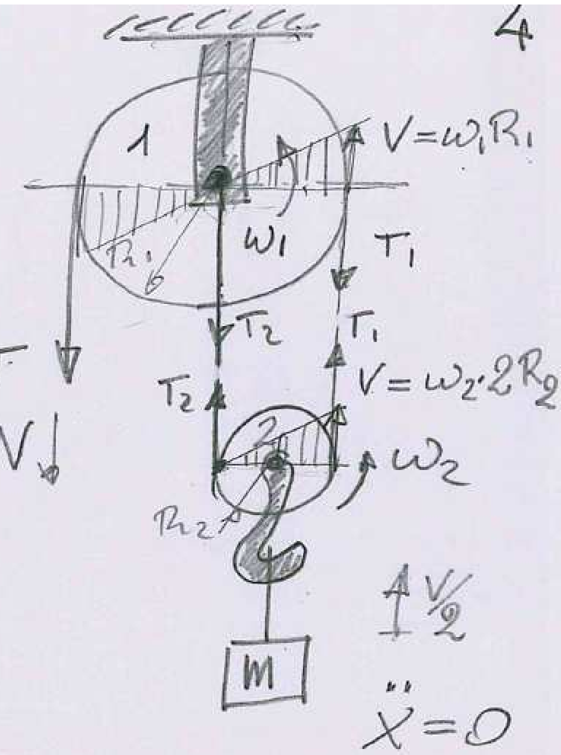
$T_1 = T_2$

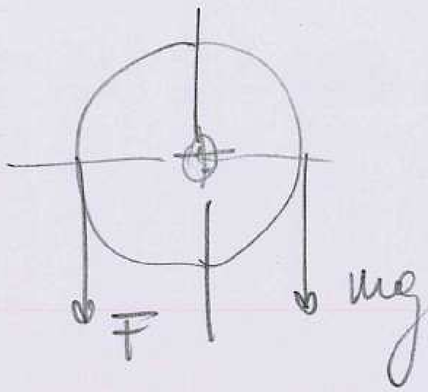
$T_1 + T_2 - mg = 0$

$2T = mg$

$T = \frac{mg}{2}$

$TV = mg \frac{V}{2}$





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$d_1 = 180\,mm$

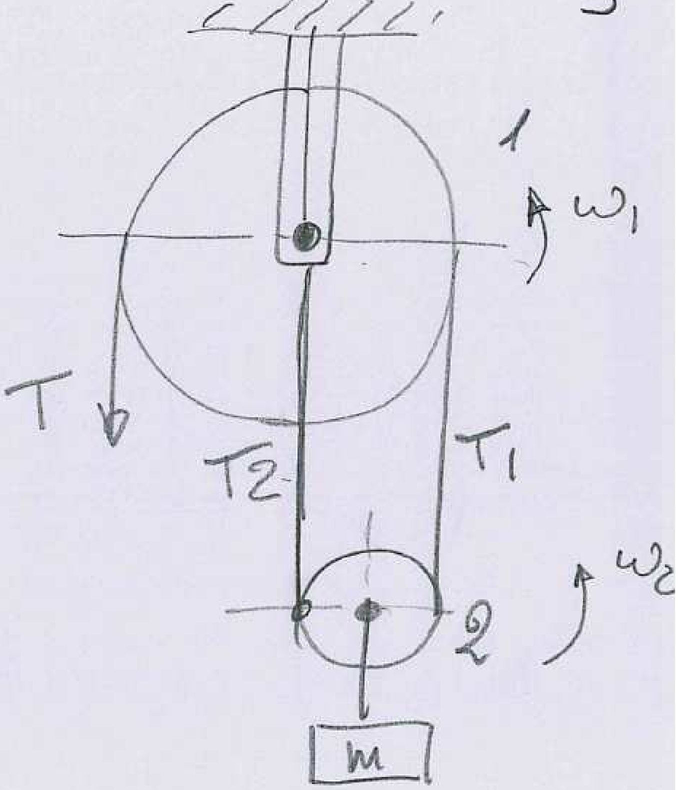
$d_2 = 90\,mm$

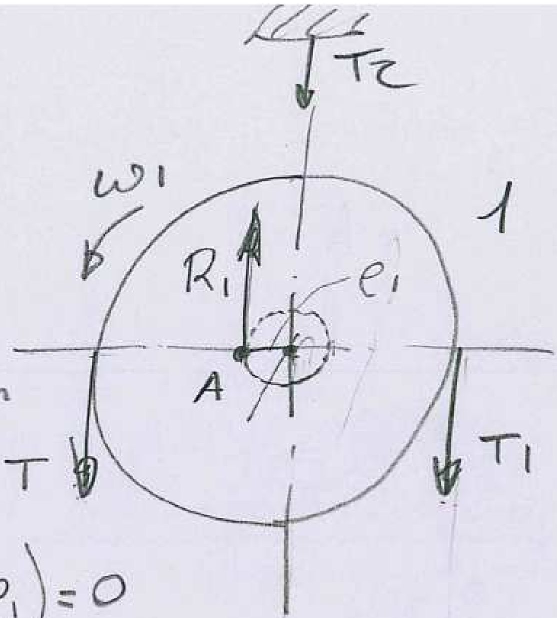
$r_1 = 10\,mm$

$r_2 = 6\,mm$

$f = 0{,}25$

$m = 200\,kg$





SALITA A V COST

$$\varphi = \operatorname{atan} f = 14.04^\circ$$

$$\rho_1 = r_1 \sin\varphi = 2{,}42\ \text{mm}$$

$$\rho_2 = r_2 \sin\varphi = 1{,}45\ \text{mm}$$

$$A)\quad T\left(\frac{d_1}{2} - \rho_1\right) - T_1\left(\frac{d_1}{2} + \rho_1\right) = 0$$

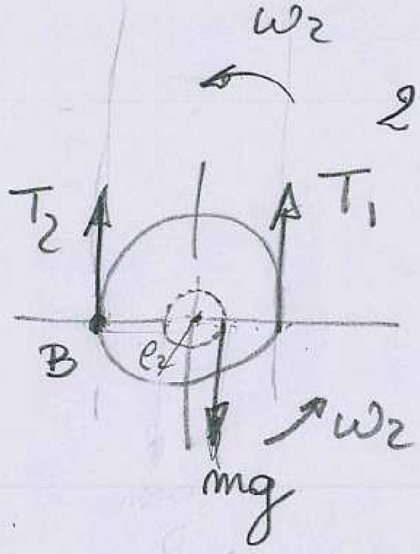
$$T_1 + T_2 - mg = 0$$

$$B)\quad T_1 d_2 - mg\left(\frac{d_2}{2} + \rho_2\right) = 0$$

$$T_1 = mg\,\frac{\frac{d_2}{2} + \rho_2}{d_2} = 1014\ N$$

$$T_2 = mg - T_1 = 948\ N$$

$$T = T_1\,\frac{\frac{d_1}{2} + \rho_1}{\frac{d_1}{2} - \rho_1} = 1072\ N$$



DISCESA A V=COST

$\varphi = 14{,}04°$

$e_1 = 2{,}42\,mm \quad e_2 = 1{,}45\,mm$

$\overset{\curvearrowleft}{A}) \; T\left(\frac{d_1}{2}+e_1\right) - T_1\left(\frac{d_1}{2}-e_1\right) = 0 \quad T$

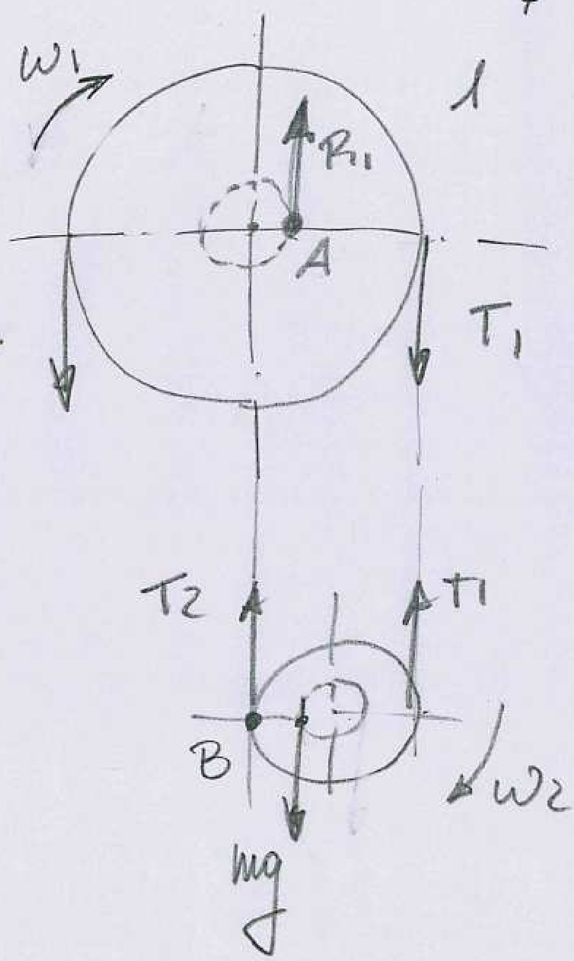
$T_1 + T_2 - mg = 0$

$\overset{\curvearrowleft}{B}) \; T_1 d_2 - mg\left(\frac{d_2}{2}-e_2\right) = 0$

$T = 897\,N$

$T_1 = 948\,N$

$T_2 = 1014\,N$



$m_c = 80\,kg$ $\quad a = 70\,cm$ $\quad M = m_c + m_b$

$m_b = 10\,kg$ $\quad b = 40\,cm$ $\quad d = 120\,cm$

$\quad c = 110\,cm$

$f = \frac{u}{2} \sim 0$ $\quad \phi_R = 29'' = 736.6\,mm$

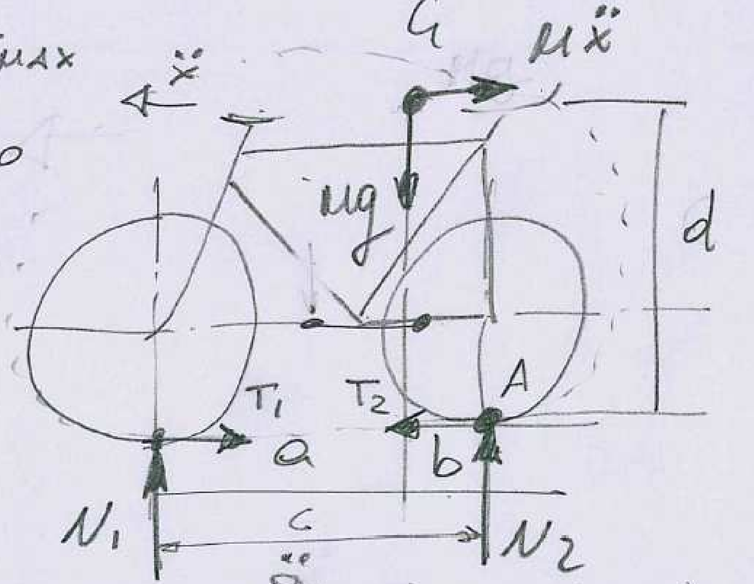
$f_{AD} = 0.6$ $\quad f_{DIN} = 0.5$ $\quad l_P = 175\,mm$

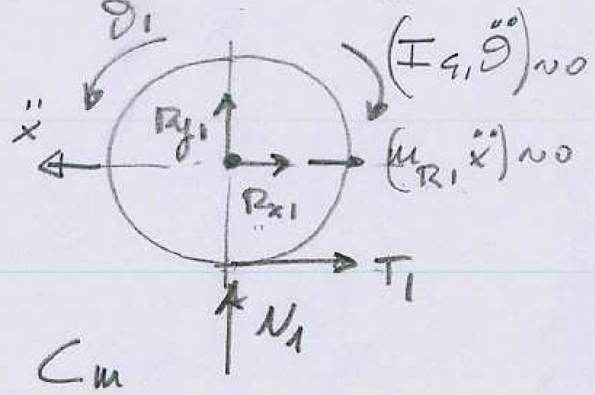
$i = \frac{\omega_1}{\omega_2} = \frac{z_2}{z_1} = \frac{40}{26} = 1.54$

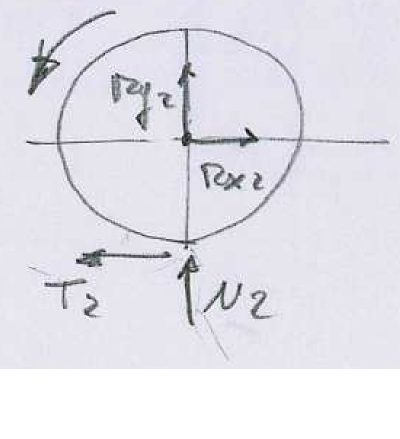
$\ddot{x}_{MAX} \quad C_{MAX} \quad F_{MAX}$

$$
\begin{cases}
A)\ -N_1 c + mgb - m\ddot{x}d = 0 \\
N_1 + N_2 - mg = 0 \\
T_2 - T_1 - m\ddot{x} = 0 \\
T_2 \leq f_{AD} N_2 \\
T_1 \frac{\phi_R}{2} = 0 \\
-T_2 \frac{\phi_R}{2} + C_m = 0
\end{cases}
$$

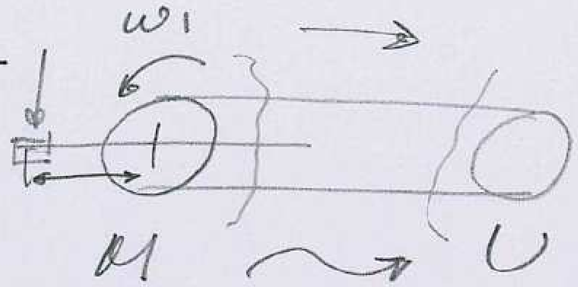
$N_1, N_2, T_2, T_1, \ddot{x}, C_m$







$\eta \sim 0{,}99$
~ 1



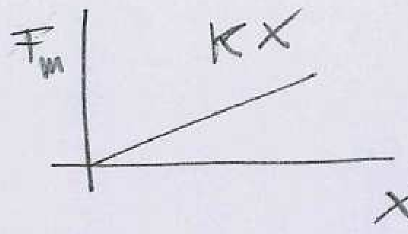
$$F \cdot l_p \cdot \omega_1 \simeq C_m \omega_2$$

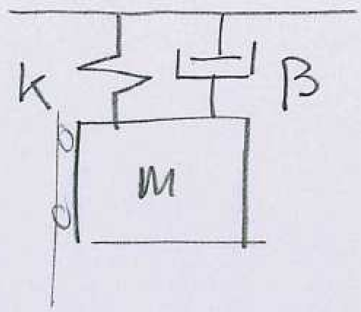
$$M_{mot} \cdot \omega_1 \simeq M_{UT}\, \omega_2$$

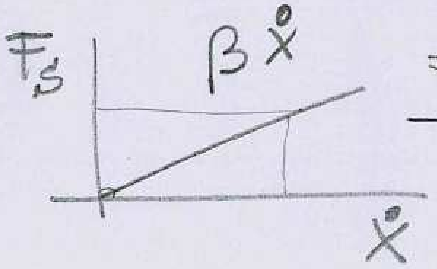
$$F l_p = C_m \frac{\omega_2}{\omega_1}$$

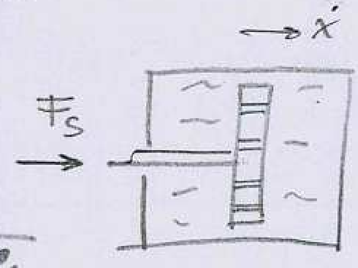
$$F = \frac{C_m}{l_p} \frac{\omega_2}{\omega_1} = \frac{C_m}{l_p} \cdot \frac{1}{i}$$

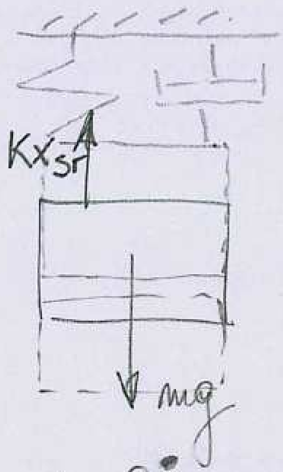
Oscillazioni ad un grado di libertà.





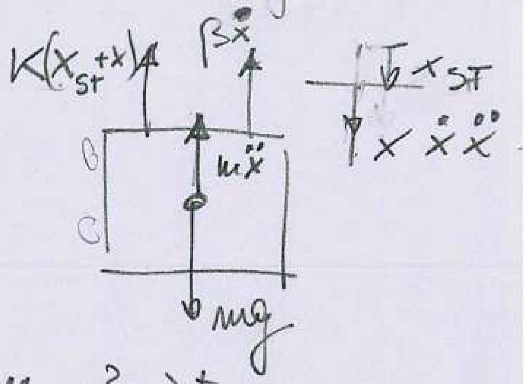






$$K x_{ST} = mg$$

$$\cancel{K x_{ST}} + Kx + \beta\dot{x} + m\ddot{x} - \cancel{mg} = 0$$



$$m\ddot{x} + \beta\dot{x} + Kx = 0$$

$$\ddot{x} + \frac{\beta}{m}\dot{x} + \frac{k}{m}x = 0$$

$$x = e^{\lambda t} \qquad \dot{x} = \lambda e^{\lambda t} \qquad \ddot{x} = \lambda^2 e^{\lambda t}$$

$$\lambda^2 + \frac{\beta}{m}\lambda + \frac{k}{m} = 0$$

$$\lambda_{1,2} = \frac{-\beta \pm \sqrt{\beta^2 - 4mk}}{2m} = -\frac{\beta}{2m} \pm \omega_n\sqrt{\zeta^2 - 1}$$

$$\omega_n = \sqrt{\frac{k}{m}} \ ; \quad \zeta = \frac{\beta}{2\sqrt{km}} = \frac{\beta}{2m\omega_n}$$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$$

$$\zeta = \frac{\beta}{2m\omega_n} \qquad \beta_{CR} = 2m\omega_n \qquad \zeta = \frac{\beta}{\beta_{CR}}$$

$$x = e^{\lambda t}$$

$$\lambda_{1,2} = \omega_n\left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)$$

$$x = a e^{\lambda_1 t} + b e^{\lambda_2 t}$$