FONDAMENTI DI MECCANICA E BIOMECCANICA [IN/0165]

Lezione del 19 ottobre 2017.

Titolo:
Analisi del moto di meccanismi.

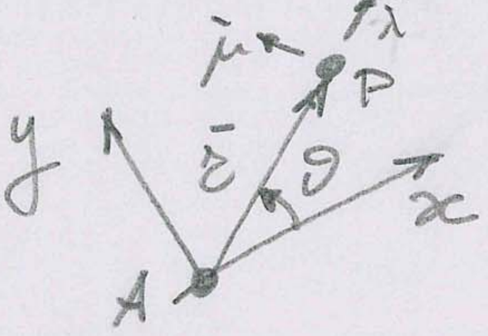
Contenuti:
Analisi del moto di meccanismi in presenza di moto relativo.
Moto di navicella a bordo di corpo girevole.
Moto di glifo oscillante.
Moto di ruota rotolante in aderenza collegata a meccanismo a glifo incernierato.

Riferimento:
Ferraresi C., Raparelli T. "Meccanica applicata - Terza edizione", CLUT, 2007.
Cap. 1 - Elementi di cinematica.
Pagg. 26 - 39
Legnani G., Palmieri G. "Fondamenti di meccanica e biomeccanica del movimento", CittàStudi, 2016.
Cap. 3 – Cinematica.
Pagg. 122 - 138

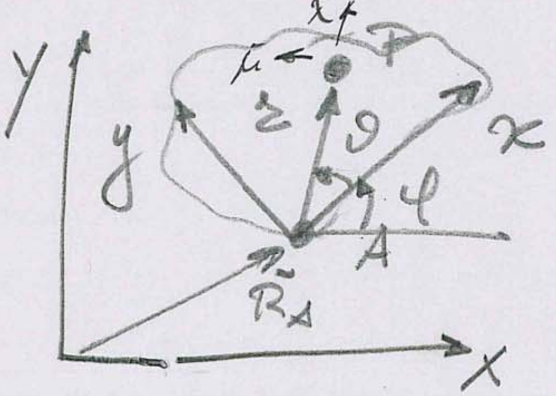
$\bar{V}_{P_r} = \dot{r}\,\bar{\lambda} + r\dot{\vartheta}\,\bar{\mu}$ RELATIVA

$\bar{V}_{P_t} = \dot{x}_A\,\vec{i} + \dot{y}_A\,\vec{j} + r\dot{\varphi}\,\mu$ TRASCINAM.

RELATIVO



TRASC



$|r| = \text{cost}$

$\vartheta = \text{cost}$

ASSOLUTA

$$\bar{V}_P = \dot{x}_A\,\vec{i} + \dot{y}_A\,\vec{j} + \dot{r}\,\bar{\lambda} + r(\dot{\varphi} + \dot{\vartheta})\,\bar{\mu}$$

$$\bar{V}_P = \bar{V}_{P_r} + \bar{V}_{P_t}$$

$$\vec{a}_P = \ddot{x}_A\vec{i} + \ddot{y}_A\vec{j} + \ddot{r}\vec{\lambda} + \dot{r}\dot{\vec{\lambda}} + \dot{r}(\dot{\varphi}+\dot{\vartheta})\bar{\mu} +$$
$$+ r(\ddot{\varphi}+\ddot{\vartheta})\bar{\mu} + r(\dot{\varphi}+\dot{\vartheta})\dot{\bar{\mu}}$$

$$\bar{a}_P = \ddot{x}_A\vec{i} + \ddot{y}_A\vec{j} + \ddot{r}\bar{\lambda} + \dot{r}(\dot{\varphi}+\dot{\vartheta})\bar{\mu} + \dot{r}(\dot{\varphi}+\dot{\vartheta})\bar{\mu} +$$
$$+ r(\ddot{\varphi}+\ddot{\vartheta})\bar{\mu} + r(\dot{\varphi}+\dot{\vartheta})^2(-\bar{\lambda})$$

- RELATIVA

$$\boxed{\bar{a}_{P_r}} = \ddot{r}\bar{\lambda} + \dot{r}\dot{\bar{\lambda}}_r + \dot{r}\dot{\vartheta}\bar{\mu} + r\ddot{\vartheta}\bar{\mu} + r\dot{\vartheta}\dot{\bar{\mu}}_r =$$
$$= \ddot{r}\bar{\lambda} + \dot{r}\dot{\vartheta}\bar{\mu} + \dot{r}\dot{\vartheta}\bar{\mu} + r\ddot{\vartheta}\bar{\mu} + r\dot{\vartheta}^2(-\bar{\lambda})$$

- TRASCINAMENTO

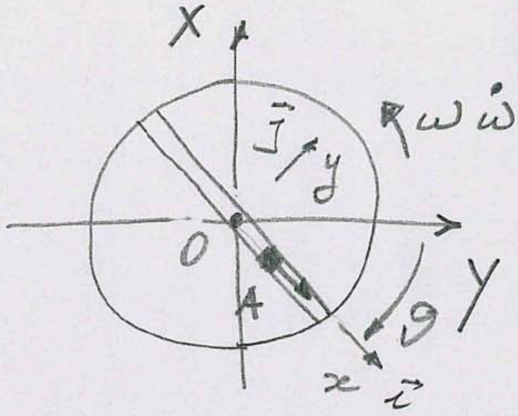
$$\bar{a}_{P_t} = \ddot{x}_A\vec{i} + \ddot{y}_A\vec{j} + \dot{r}\dot{\varphi}\bar{\mu} + r\ddot{\varphi}\bar{\mu} + r\dot{\varphi}\dot{\bar{\mu}}_t =$$
$$= \ddot{x}_A\vec{i} + \ddot{y}_A\bar{j} + \underbrace{\dot{r}}_{=0}\dot{\varphi}\bar{\mu} + r\ddot{\varphi}\bar{\mu} + r\dot{\varphi}^2(-\bar{\lambda})$$

- ASSOLUTA

$$\bar{a}_P = \ddot{x}_A\vec{i} + \ddot{y}_A\vec{j} + \boxed{\ddot{r}\vec{\lambda}} + 2\dot{r}\dot{\varphi}\bar{\mu} + \boxed{2\dot{r}\dot{\vartheta}\bar{\mu}} +$$
$$+ r\ddot{\varphi}\bar{\mu} + \boxed{r\ddot{\vartheta}\bar{\mu}} - r\dot{\varphi}^2\lambda - \boxed{r\dot{\vartheta}^2\bar{\lambda}} - 2r\dot{\varphi}\dot{\vartheta}\lambda =$$
$$= a_{P_t} + a_{P_r} + 2\dot{r}\dot{\varphi}\bar{\mu} - 2r\dot{\varphi}\dot{\vartheta}\bar{\lambda}$$

$$\bar{a}_c = 2\,\bar{\omega}_t \wedge \bar{V}_r = 2\,\dot{\varphi}\bar{k}\wedge(\dot{r}\bar{\lambda} + r\dot{\vartheta}\bar{\mu}) =$$
$$= 2\dot{r}\dot{\varphi}\,\bar{k}\wedge\bar{\lambda} + 2r\dot{\varphi}\dot{\vartheta}\,\bar{k}\wedge\bar{\mu} = 2\dot{r}\dot{\varphi}\bar{\mu} - 2r\dot{\varphi}\dot{\vartheta}\bar{\lambda}$$

$$\bar{a}_P = \vec{a}_{P_r} + \vec{a}_{P_t} + \bar{a}_c$$



$\omega = 4\ rad/s$

$\dot{\omega} = -10\ rad/s^2$

$OA = r = 150\ mm$

$\dot{r} = 125\ mm/s$

$\ddot{r} = 2025\ mm/s^2$

$$\vec{V}_A = \vec{V}_{Ar} + \vec{V}_{At}$$

$$\vec{V}_{Ar} = \dot{r}\vec{i} = 0{,}125\vec{i}\ m/s$$

$$\vec{V}_{At} = \vec{V}_{A/0} = \omega r \vec{J} = 0{,}6\vec{J}\ m/s$$

$$\bar{V}_A = 0{,}125\vec{i} + 0{,}6\vec{J}$$

$$|\bar{V}_A| = \sqrt{0{,}125^2 + 0{,}6^2} = 0{,}613\ m/s$$

$$\bar{a}_A = \vec{a}_{Ar} + \vec{a}_{At} + \vec{a}_{Ac}$$

$$\vec{a}_{Ar} = \ddot{r}\vec{i} = 2025\vec{i}\ \frac{m}{s^2}$$

$$\vec{a}_{At} = \omega^2 r(-\vec{i}) + \dot{\omega} r \vec{J} = -2{,}4\vec{i} - 1{,}5\vec{J}$$

$$\vec{a}_{Ac} = 2\vec{\omega}_t \wedge \vec{V}_r = 2\omega\bar{k} \wedge \dot{r}\vec{i} = 2\omega\dot{r}\,\bar{k}\wedge\bar{i} =$$

$$= 2\omega\dot{r}\vec{J} = 1\vec{J}$$

$$\vec{a}_A = 2{,}025\vec{i} - 2{,}4\vec{i} - 1{,}5\vec{J} + 1\vec{J} =$$

$$= -0{,}375\vec{i} - 0{,}5\vec{J}\ m/s^2$$

$a = 500\,mm$

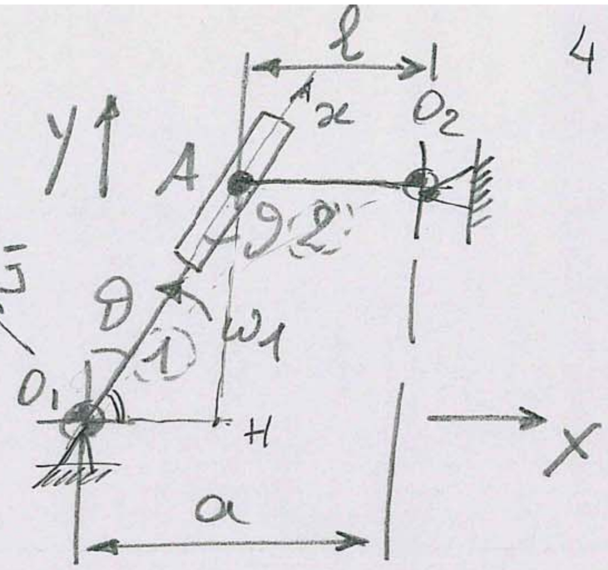
$\ell = 250\,mm$

$\vartheta = 30°$

O_2A ORIZZONTALE

$\omega_1 = 5\,rad/s$ COST

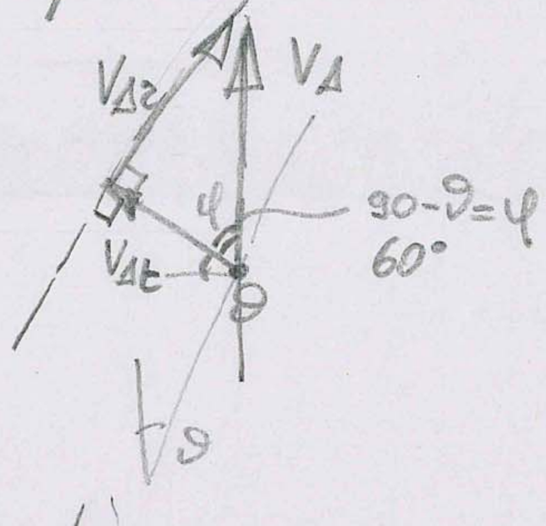
$V_{Ar}, \omega_2, a_{Ar}, \dot{\omega}_2$ -



$O_1H = a - \ell = 250\,mm$

$r = O_1A = O_1H/\sin\vartheta = 0{,}25/\sin 30 = 0{,}5\,m$

$\vec{V}_A$ =	$\vec{V}_{Ar}$ +	$\vec{V}_{At}$	
$\omega_2 \ell$?	?	$\omega_1 r$ 2,5 m/s	M
$\perp O_2A$	$// O_1A$	$\perp O_1A$	D
?	?	↖	V



$V_A = V_{At}/\cos\varphi = 2{,}5/\cos 60° = 5\,m/s$

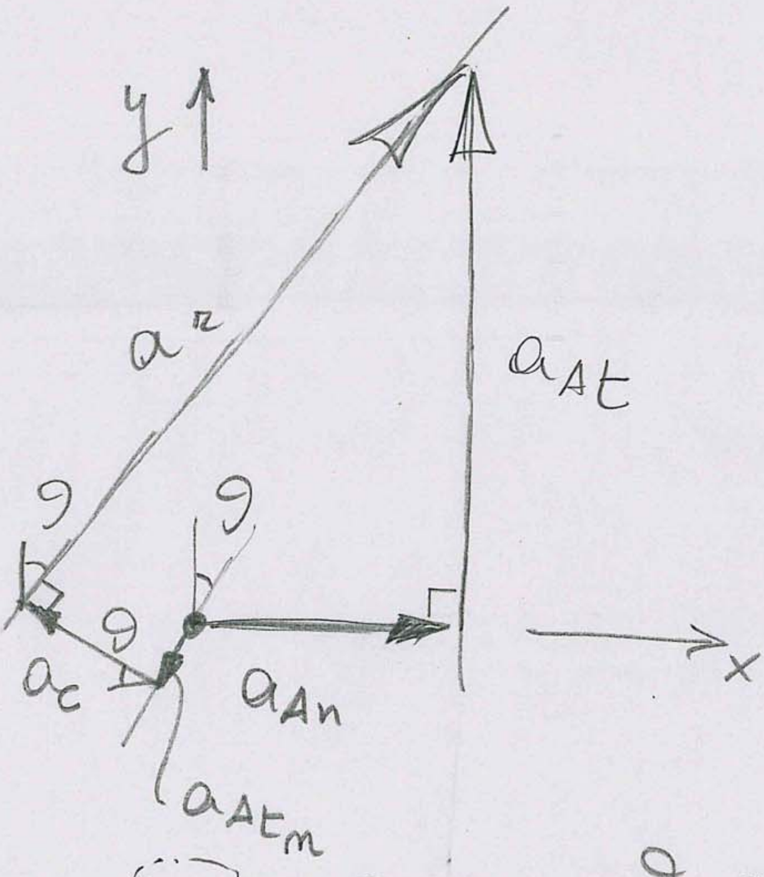
$V_{Ar} = V_{At}\,tg\,\varphi = 2{,}5 \cdot tg\,60° = 4{,}33\,m/s$

$\omega_2 = \frac{V_A}{\ell} = \frac{5}{0{,}25} = 20\,\frac{rad}{s}$; $\vec{\omega}_2 = \omega_2(-\bar{k})$

$$\bar{a}_A = \bar{a}_{Ar} + a_{At_n} + a_{At_t} + \bar{a}_c = a_{An} + a_{At}$$

	$\bar{a}_{Ar}$	a_{At_n}	a_{At_t}	$\bar{a}_c$	a_{An}	a_{At}
M	?	$\omega_1^2 r$ 12,5 m/s^2	$\dot{\omega}_1 r$ 0	43,3 m/s^2	$\omega_2^2 l$ 100 m/s^2	$\dot{\omega}_2 l$?
D	$//AO_1$	$//AO_1$	$\forall$	$\perp O_1A$	$//AO_2$	$\perp AO_2$
V	?	$A \to O_1$	$\forall$	$J\nwarrow$	$A \to O_2$	?

$$\vec{a}_c = 2\bar{\omega}_t \wedge \bar{V}_r = 2\bar{\omega}_1 \wedge \bar{V}_r = 2 \cdot 5 \cdot 4,33\ \bar{J} = 43,3\ \bar{J}\ \frac{m}{s^2}$$



x) $a_{An} - a_{Ar}\sin\vartheta + a_c\cos\vartheta + a_{At_n}\sin\vartheta = 0$

y) $a_{At} - a_{Ar}\cos\vartheta - a_c\sin\vartheta + a_{At_n}\cos\vartheta = 0$

$a_{Ar} = 287,5\ m/s^2$; $\dot{\omega}_2 = \frac{a_{At}}{l} = 1039,2\ \frac{rad}{s^2}$; $\bar{\omega}_2 = \omega_2(-$