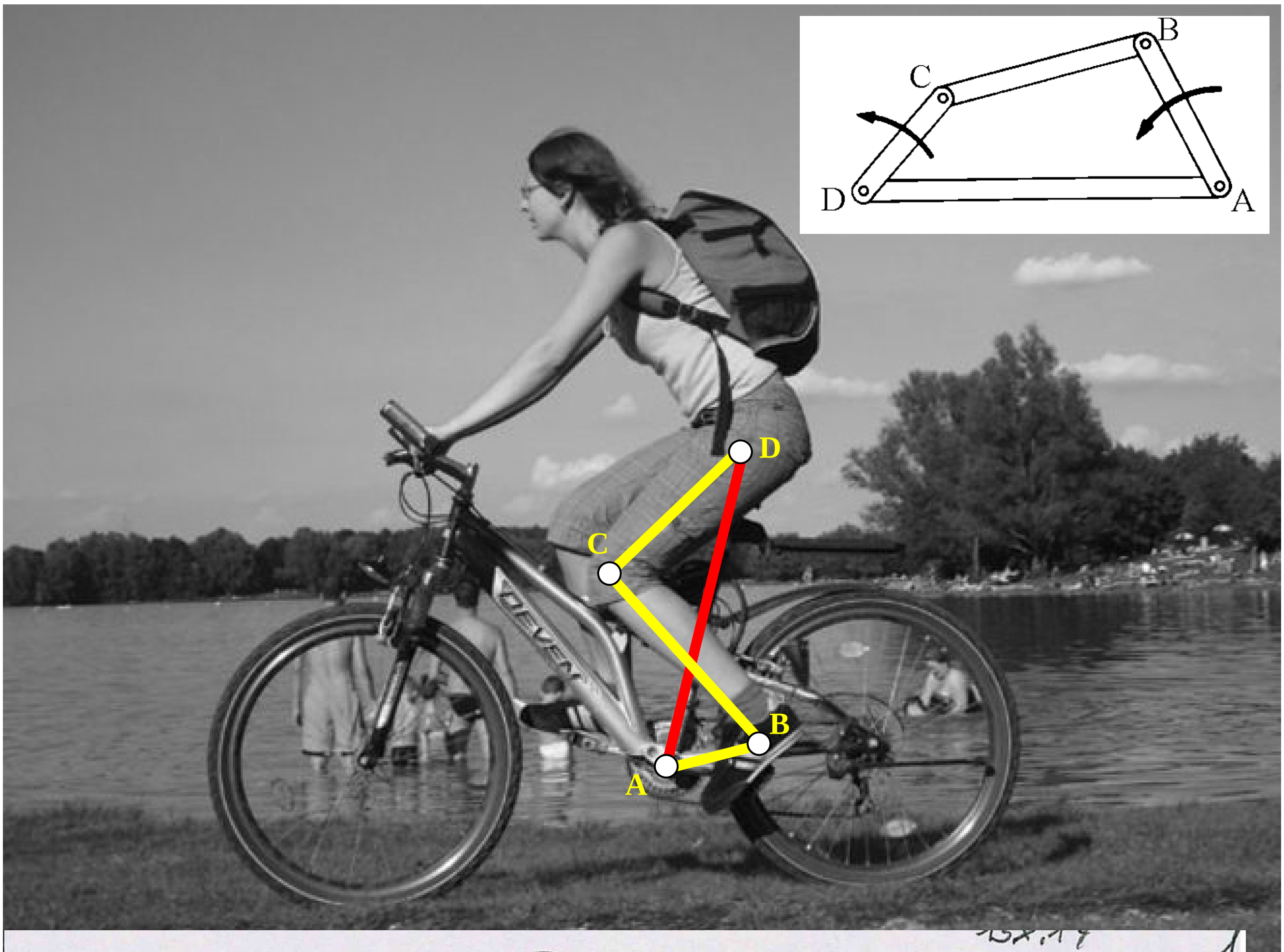
FONDAMENTI DI MECCANICA E BIOMECCANICA [IN/0165]

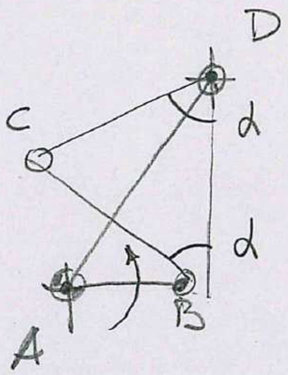
Lezione del 12 ottobre 2017.

Titolo:
Analisi di meccanismi articolati e teoria dei moti relativi.

Contenuti:
Discussione e proposta di analisi del movimento della pedalata di un ciclista, in una particolare condizione, assimilata al moto di un quadrilatero articolato.
Analisi cinematica di meccanismo biella-manovella, con biella prolungata oltre la testa e manovella ortogonale alla direzione del moto del piede di biella. Calcolo della distribuzione di velocità con la formula di Galileo. Calcolo della distribuzione delle accelerazioni con il teorema di Rivals.
Teoria di moti relativi, velocità assoluta, relativa e di trascinamento; accelerazione assoluta, relativa, di trascinamento e complementare o di Coriolis.

Riferimento:
Ferraresi C., Raparelli T. "Meccanica applicata - Terza edizione", CLUT, 2007.
Cap. 1 - Elementi di cinematica.
Pagg. 26 - 29
Legnani G., Palmieri G. "Fondamenti di meccanica e biomeccanica del movimento", CittàStudi, 2016.
Cap. 3 – Cinematica.
Pagg. 111 - 138





$\alpha = 45^\circ$ $H = 1{,}70\,m$

$AB = 200\,mm$

$CD = 0{,}245H = 0{,}42\,m$

$CB = 0{,}245H = 0{,}42\,m$

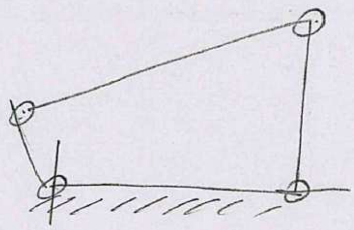
AB ORIZZONTALE

$\phi_{RUOTA} = 26''$

$CIRC._{RUOTA} = 2026\,mm$

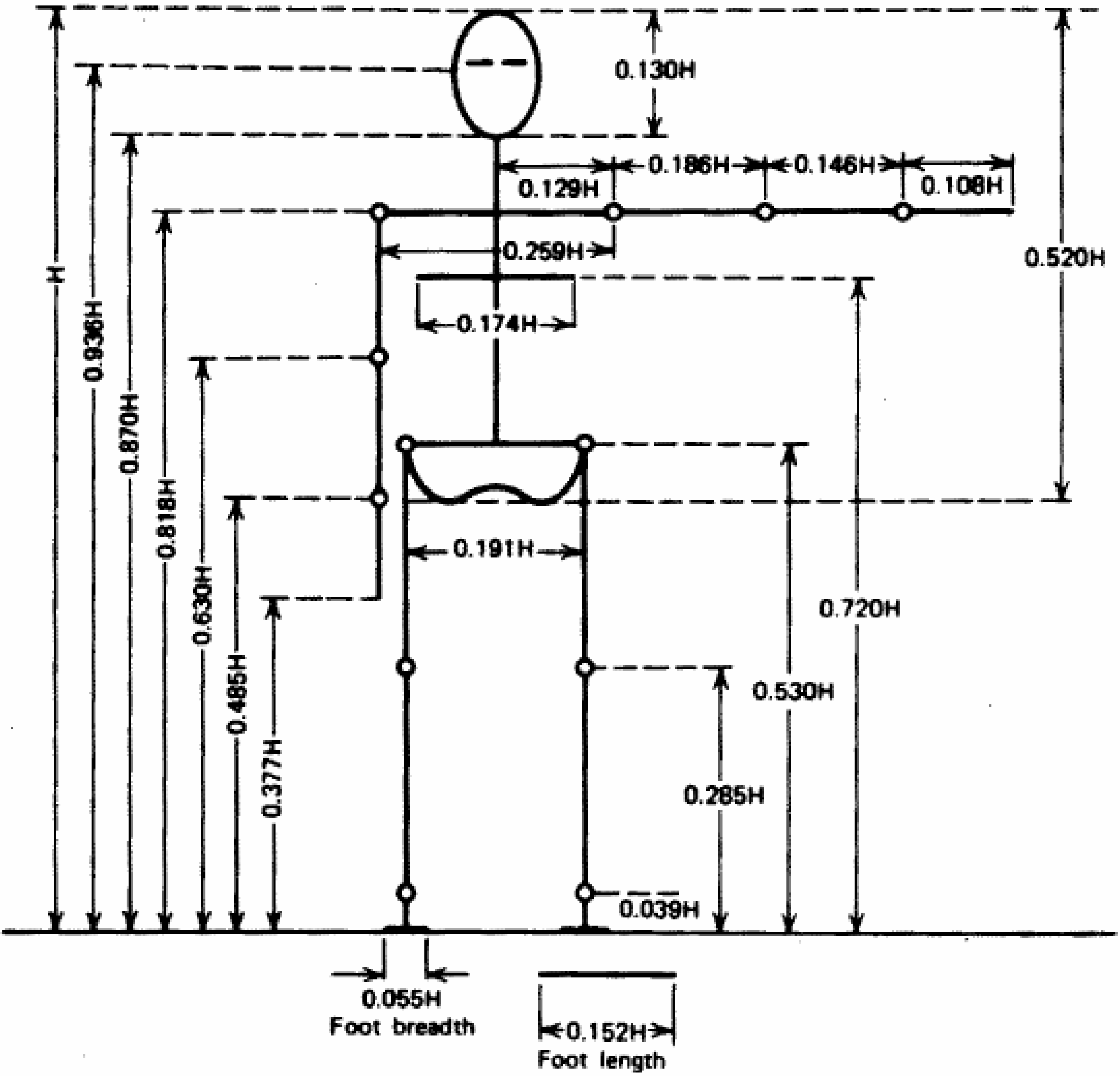
$V = 20\,km/h = COST$

$$i = \frac{\omega_{PEDALI}}{\omega_{RUOTA}} = \frac{1}{1{,}5}$$



$\omega_{BC}\,,\ \omega_{CD}$

$\dot{\omega}_{BC}\,,\ \dot{\omega}_{CD}$



Schema antropometrico, dimensioni parametrizzate con la statura H.

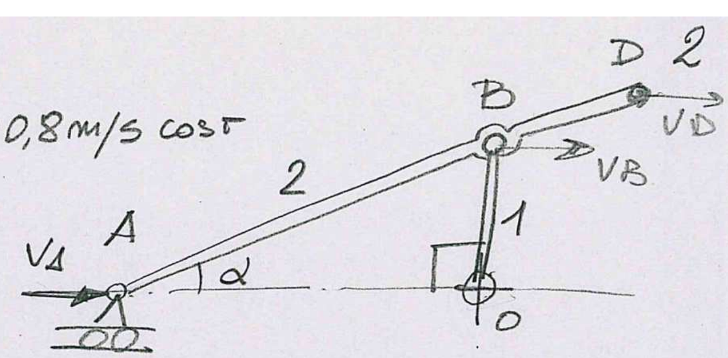
Dempster e Gaughran, *Properties of Body Segments Based on Size and Weight*, Department of Anatomy, The University of Michigan, Ann Arbor, Michigan and Department of Anatomy, The Ohio State University, Columbus, 1967.

$AB = 1000$ mm

$BD = 500$ mm

$OB = 500$ mm

$V_A = 0{,}8$ m/s cost



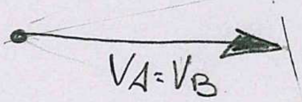
$V_D \quad \omega_{OB} \quad \omega_{AD}$

$a_B \quad \dot{\omega}_{AD}$

$$\alpha = \arcsin \frac{OB}{AB} = 30°$$

$$\bar{V}_B = \bar{V}_A + \bar{V}_{B/A}$$

$\bar{V}_B$	$\bar{V}_A$	$\bar{V}_{B/A}$	
$\omega_1 OB$?	0,8 m/s	$\omega_2 AB$?	M
$\perp OB$	// AO	$\perp AB$	D
?	→	?	V

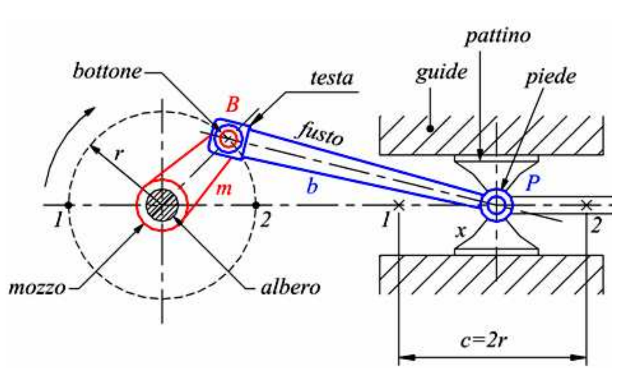


$V_{B/A} = 0 \quad \omega_2 = 0$

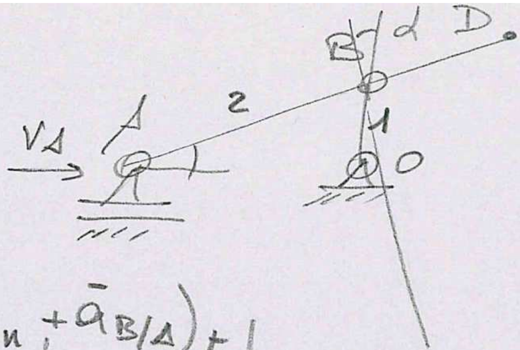
$V_A = V_B = V_D$

$$\omega_1 = \frac{V_B}{OB} = \frac{0{,}8}{0{,}5} = 1{,}6 \frac{rad}{s}$$

$$\bar{\omega}_1 = \omega_1 (-k)$$



$\bar{a}_A = \bar{0}$



$\bar{a}_B = \bar{a}_{Bn} + \bar{a}_{Bt}$

$\bar{a}_{Bn} + \bar{a}_{Bt} = \bar{a}_A + \bar{a}_{B/A})_n + \bar{a}_{B/A})_t$

$\omega_1^2 OB$ m/s² 1,28	$\dot{\omega}_1 OB$?	0	$\omega_2^2 AB$ 0	$\dot{\omega}_2 AB$?	M
// OB	⊥ OB	∀	∀	⊥ AB	D
B→O	?	∀	∀	?	V

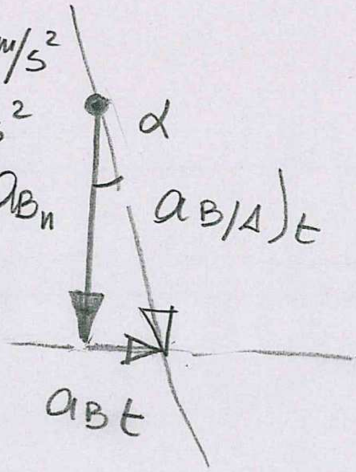
$a_{B/A})_t = a_{Bn} / \cos\alpha = 1{,}28 / \cos 30° = 1{,}48\ m/s^2$

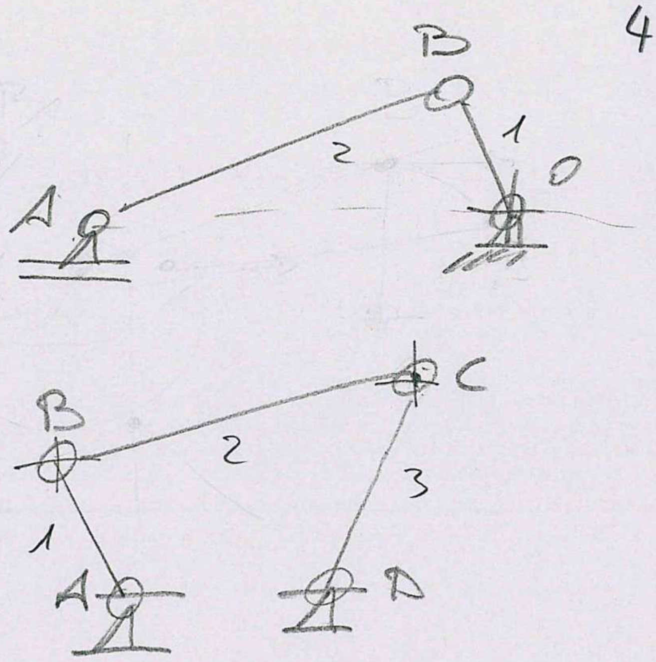
$a_{Bt} = a_{Bn}\, tg\,\alpha = 1{,}28 \cdot tg\,30° = 0{,}74\ m/s^2$

$\dot{\omega}_2 = \dfrac{a_{B/A})_t}{AB} = \dfrac{1{,}48}{1} = 1{,}48\ \dfrac{rad}{s^2}$

$\bar{\dot{\omega}}_2 = \dot{\omega}_2 (-k)$

$a_B = \sqrt{a_{Bn}^2 + a_{Bt}^2} = 1{,}48\ m/s^2$





NELLE CERNIERE
NON SI HA MOTO RELATIVO

Teoria dei moti relativi

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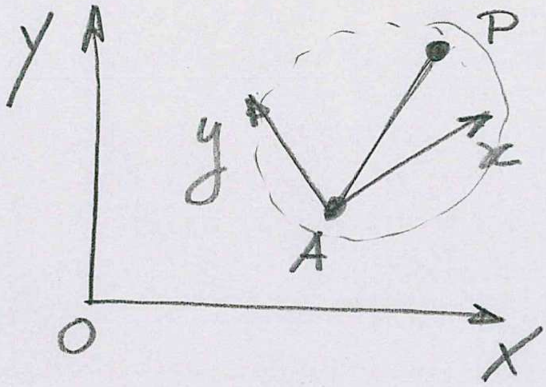
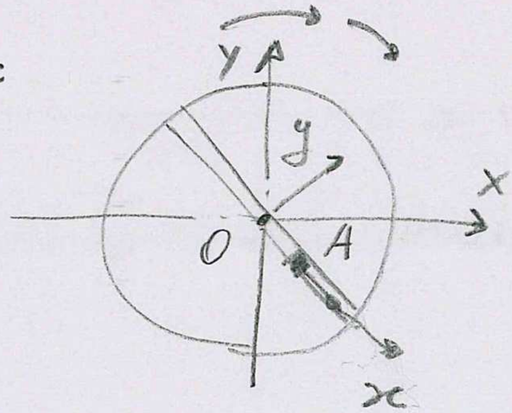


$$\bar{V}_P = \bar{V}_{Pr} + \bar{V}_{Pt}$$

$$\bar{V}_{Pt} = \bar{V}_A + \bar{V}_{P/A}$$

$$\vec{a}_P = \vec{a}_{Pr} + \vec{a}_{Pt} + \vec{a}_c$$

$$\vec{a}_c = 2\,\bar{\omega}_t \wedge \vec{V}_r$$



Nel seguito si ricaveranno le espressioni di velocità ed accelerazione assolute, relative, di trascinamento e complementare.

Si esprime la posizione del punto P e si ricavano velocità ed accelerazione derivando.

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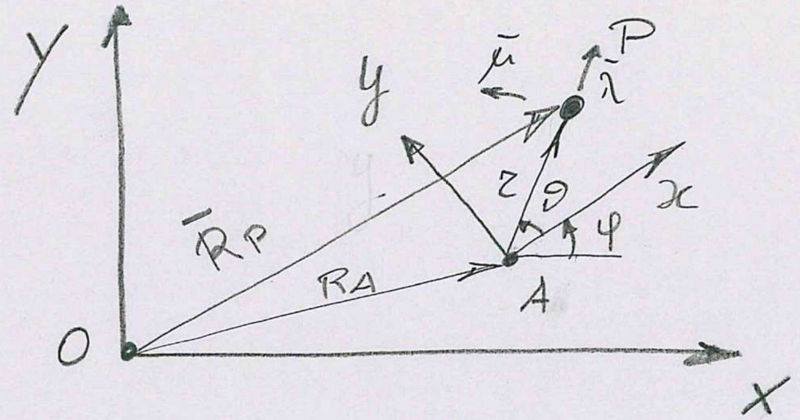


$$\vec{R}_P = \vec{R}_A + \vec{r} = x_A\vec{i} + y_A\vec{j} + r\bar{\lambda}$$

$$\bar{V}_P = \frac{d\bar{R}_P}{dt} = \dot{x}_A\vec{i} + \dot{y}_A\vec{j} + \dot{r}\bar{\lambda} + r\dot{\bar{\lambda}}$$

$$\bar{a}_P = \frac{d^2R_P}{dt^2} = \ddot{x}_A\vec{i} + \ddot{y}_A\vec{j} + \ddot{r}\bar{\lambda} + \dot{r}\dot{\bar{\lambda}} + \dot{r}\dot{\bar{\lambda}} + r\ddot{\bar{\lambda}}$$

$$\dot{\bar{\lambda}} = (\dot{\varphi} + \dot{\vartheta})\bar{k} \wedge \bar{\lambda} = (\dot{\varphi} + \dot{\vartheta})\bar{\mu}$$

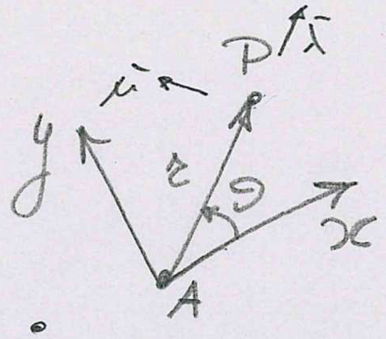
$$\ddot{\bar{\lambda}} = \frac{d\dot{\bar{\lambda}}}{dt} = (\ddot{\varphi} + \ddot{\vartheta})\bar{\mu} + (\dot{\varphi} + \dot{\vartheta})\dot{\bar{\mu}}$$

$$\dot{\bar{\mu}} = (\dot{\varphi} + \dot{\vartheta})\bar{k} \wedge \bar{\mu} = (\dot{\varphi} + \dot{\vartheta})(-\bar{\lambda})$$

$$\ddot{\bar{\lambda}} = (\ddot{\varphi} + \ddot{\vartheta})\bar{\mu} + (\dot{\varphi} + \dot{\vartheta})^2(-\bar{\lambda})$$

$$\bar{V}_P = \dot{x}_A \vec{i} + \dot{y}_A \bar{j} + \dot{r}\bar{\lambda} + r(\dot{\varphi} + \dot{\theta})\bar{\mu}$$

VELOCITÀ RELATIVA



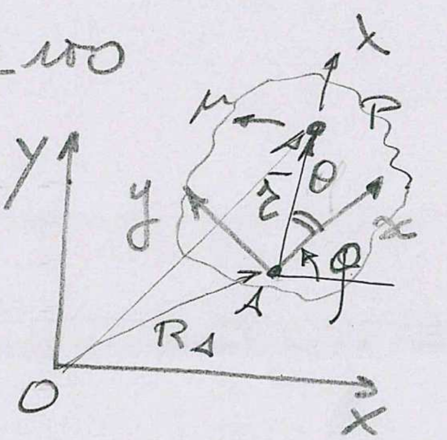
$$\bar{AP} = r\bar{\lambda}$$

$$\bar{V}_{P_r} = \dot{r}\bar{\lambda} + r\dot{\bar{\lambda}}_r$$

$$\bar{V}_{P_r} = \dot{r}\bar{\lambda} + r\dot{\vartheta}\bar{\mu}$$

$$\dot{\lambda}_r = \dot{\vartheta}\bar{k} \wedge \bar{\lambda} = \dot{\vartheta}\bar{\mu}$$

VELOCITÀ TRASCINAMENTO



$$\bar{OP} = \bar{R}_A + r\bar{\lambda}$$

$$V_{P_t} = \dot{x}_A \vec{i} + \dot{y}_A \vec{j} + \underbrace{\dot{r}}_{=0}\bar{\lambda} + r\dot{\bar{\lambda}}_t$$

$$V_{P_t} = \dot{x}_A \vec{i} + \dot{y}_A \vec{j} + r\dot{\varphi}\bar{\mu}$$

$$|r| = \text{cost}$$

$$\theta = \text{cost}$$

$$\dot{\lambda}_t = \dot{\varphi}\bar{k} \wedge \bar{\lambda} = \dot{\varphi}\bar{\mu}$$

$$\bar{V}_P = \bar{V}_{P_t} + \bar{V}_{P_r}$$

$$\bar{a}_P = \ddot{x}_A \vec{i} + \ddot{y}_A \bar{j} + \ddot{r} \lambda + 2\dot{r}(\dot{\varphi} + \dot{\vartheta}) \bar{\mu} +$$
$$+ r\left[(\ddot{\varphi} + \ddot{\vartheta}) \bar{\mu} - (\dot{\varphi} + \dot{\vartheta})^2 \bar{\lambda}\right]$$

RELATIVA

$$\bar{a}_{Pr} = \frac{d\bar{V}_{Pr}}{dt} = \ddot{r}\bar{\lambda} + \dot{r}\dot{\lambda}_{rel} + \dot{r}\dot{\vartheta}\bar{\mu} + r\ddot{\vartheta}\bar{\mu} + r\dot{\vartheta}\dot{\bar{\mu}}_{rel} =$$
$$= \ddot{r}\bar{\lambda} + \dot{r}\dot{\vartheta}\bar{\mu} + \dot{r}\dot{\vartheta}\bar{\mu} + r\ddot{\vartheta}\bar{\mu} - r\dot{\vartheta}^2 \lambda$$

TRASCINAMENTO $(r, \vartheta \text{ cost})$

$$\bar{a}_{Pt} = \frac{dV_{Pt}}{dt} = \ddot{x}_A \vec{i} + \ddot{y}_A \bar{j} + \underbrace{\dot{r}}_{=0}\dot{\varphi}\bar{\mu} + r\ddot{\varphi}\bar{\mu} + r\dot{\varphi}\dot{\mu}_t$$
$$= \ddot{x}_A \vec{i} + \ddot{y}_A \vec{j} + r\ddot{\varphi}\bar{\mu} - r\dot{\varphi}^2 \lambda$$

DUNQUE

$$\bar{a}_P = \ddot{x}_A + \ddot{y}_A \bar{j} + \ddot{r}\bar{\lambda} + 2\dot{r}\dot{\varphi}\bar{\mu} + 2\dot{r}\dot{\vartheta}\bar{\mu} +$$
$$+ r\ddot{\varphi}\bar{\mu} + r\ddot{\vartheta}\bar{\mu} - r\dot{\varphi}^2\bar{\lambda} - r\dot{\vartheta}^2\bar{\lambda} - 2r\dot{\varphi}\dot{\vartheta}\bar{\lambda} =$$
$$= \bar{a}_{Pt} + \bar{a}_{Pr} + 2\dot{r}\dot{\varphi}\bar{\mu} - 2r\dot{\varphi}\dot{\vartheta}\bar{\lambda} =$$
$$= \bar{a}_{Pt} + \bar{a}_{Pr} + \bar{a}_C$$

$$a_C = 2\dot{\varphi}\bar{k} \wedge \bar{v}_r = 2\dot{\varphi} k \wedge (\dot{r}\bar{\lambda} + r\dot{\vartheta}\bar{\mu}) =$$
$$= 2\dot{r}\dot{\varphi}\bar{\mu} - 2r\dot{\vartheta}\dot{\varphi}\bar{\lambda}$$