

## **FONDAMENTI DI MECCANICA E BIOMECCANICA [IN/0165]**

**Lezione del 10 novembre 2017.**

### **Titolo:**

Attrito statico e dinamico, esempi di accoppiamenti di superfici coniugate in presenza di attrito e aderenza.

### **Contenuti:**

Attrito statico e dinamico, esempi di accoppiamenti di superfici coniugate in presenza di attrito e aderenza.

Perno ad attrito secco, forze scambiate, forze di attrito, momento resistente al moto dovuto all'attrito.

Attrito volvente; valutazione delle azioni dissipatore in avanzamento di mezzo ferroviario su rotaia e mezzo stradale su ruota pneumatica.

Analisi del moto di mezzo interagente con ista in presenza di attrito.

Analisi del moto di corpo su piano inclinato: corpo strisciante, corpo in rotolamento puro e corpo rotante e strisciante.

Applicazione del teorema dell'energia a corpi in moto in presenza di attrito.

### **Riferimento:**

Ferraresi C., Raparelli T. "Meccanica applicata - Terza edizione", CLUT, 2007.

Cap. 3 – Attrito.

Pagg. 93 - 114

Legnani G., Palmieri G. "Fondamenti di meccanica e biomeccanica del movimento", CittàStudi, 2016.

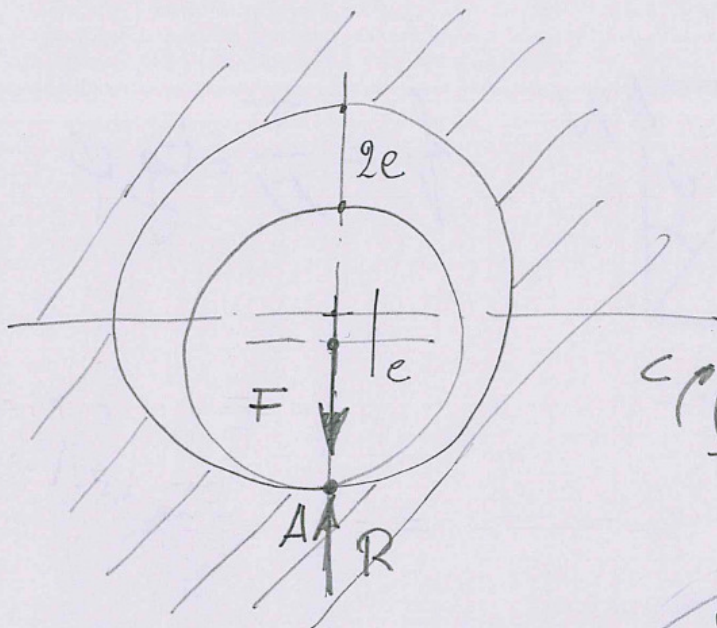
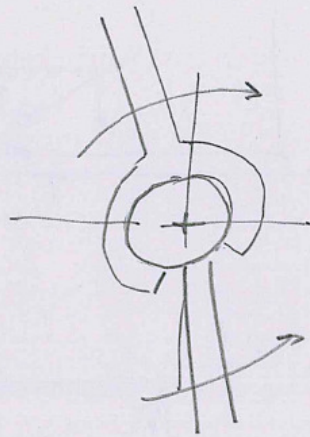
Cap. 4.4 – Forze di attrito e dissipative.

Pagg. 175 - 184

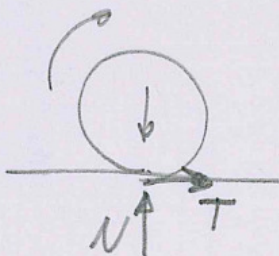
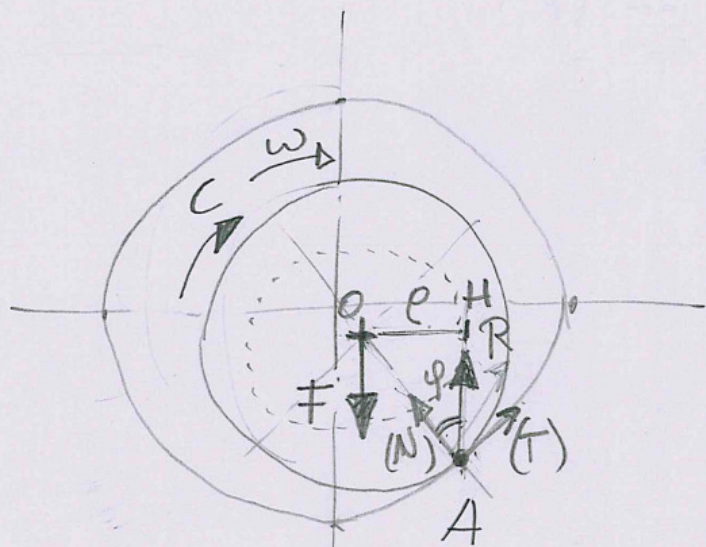
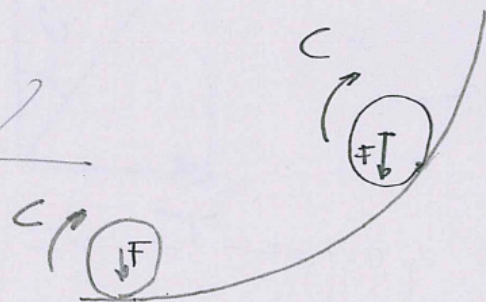
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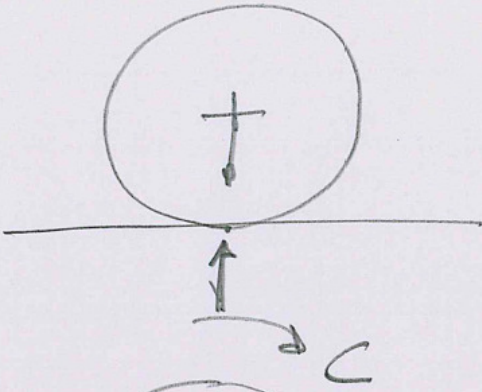
# ACCOPPIAMENTO PERNO - BOCCOLA



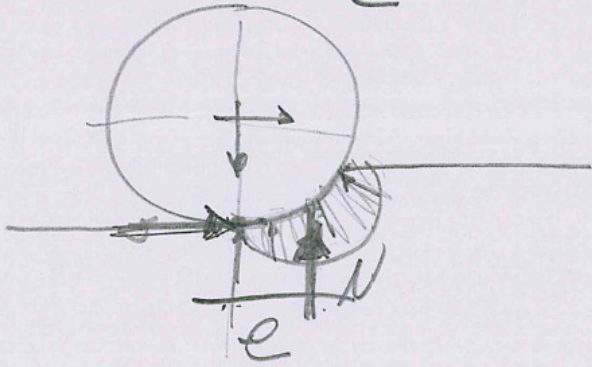
$$\bar{R} - \bar{F} = \bar{0}$$



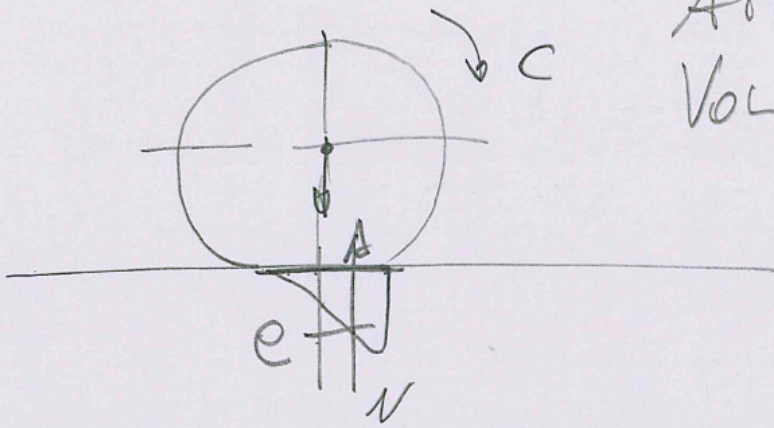
$$\left. \begin{aligned} \bar{F} - \bar{R} &= \bar{0} \\ R \cdot e - C &= 0 \\ e &= OA \cdot \sin \varphi = r_p \sin \varphi \\ C &= F r_p \sin \varphi \quad \varphi = \arctan f \end{aligned} \right\}$$



$P \approx 1000 \text{ CV}$   
15 VAGONI

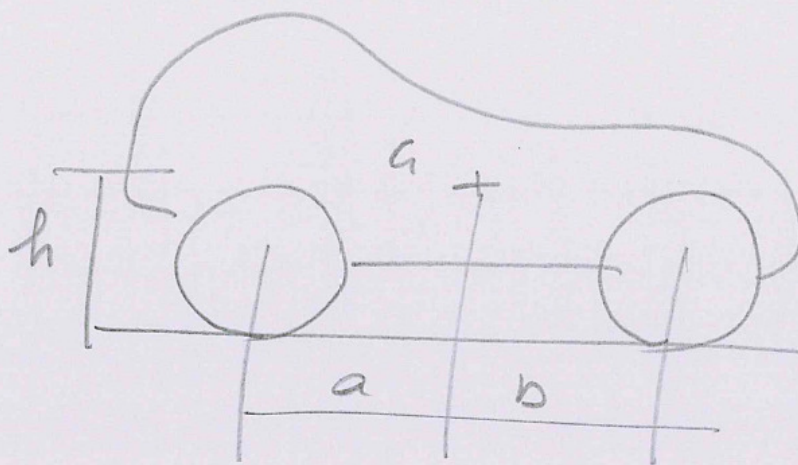


$P \approx 500 \text{ CV}$   
1 RIMORCHIO  
N 1 VAGONE



ATTORNO  
VOLVENTE

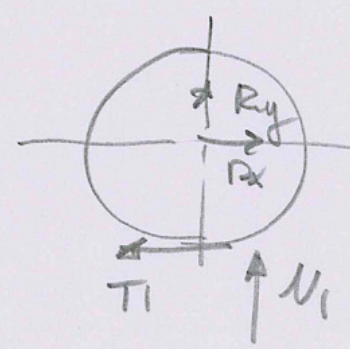
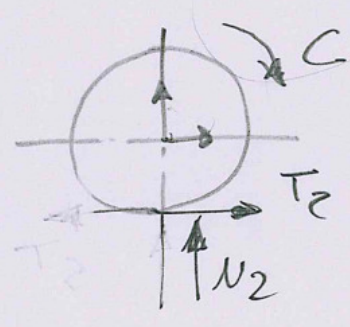
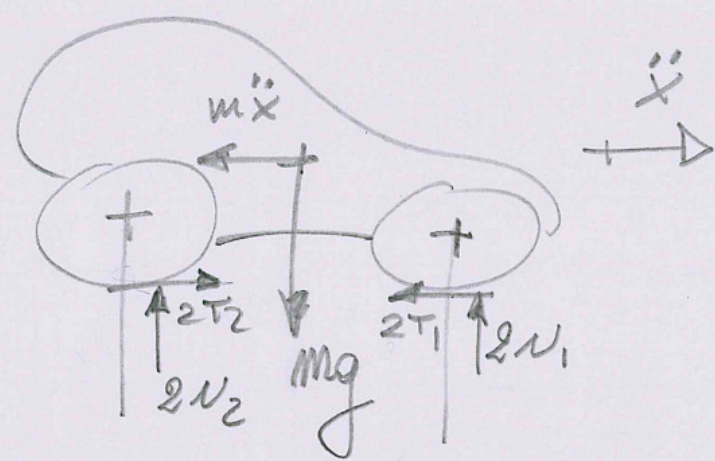




$M = 1000 \text{ kg}^3$

- $a = 1 \text{ m}$
- $b = 1,5 \text{ m}$
- $h = 0,5 \text{ m}$
- $C_H = 100 \text{ N/m}$
- $e = 2 \text{ cm}$

Auto  
TR 12, Post,



$$M = 100 \text{ kg}$$

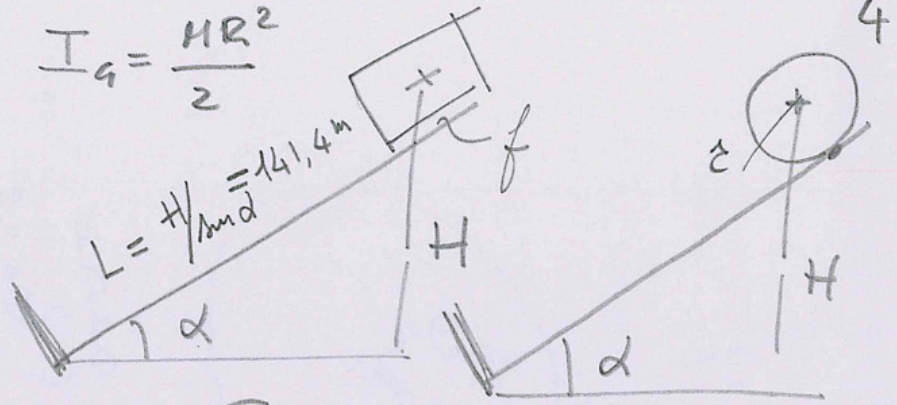
$$H = 100 \text{ m}$$

$$f = 0,1$$

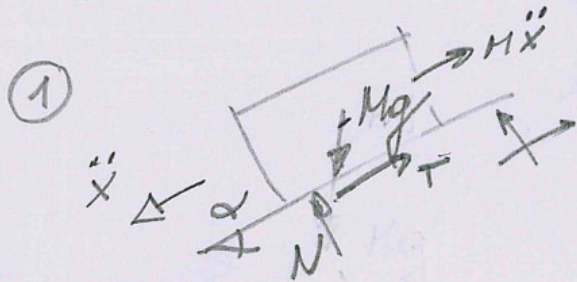
$$\alpha = 45^\circ$$

$$r = 0,25 \text{ m}$$

$$I_G = \frac{MR^2}{2}$$



t, V ARRIVO



$$\begin{cases} T - Mg \sin \alpha + M \ddot{x} = 0 \\ N - Mg \cos \alpha = 0 \\ T = f N \end{cases}$$

$$f Mg \cos \alpha - Mg \sin \alpha + M \ddot{x} = 0$$

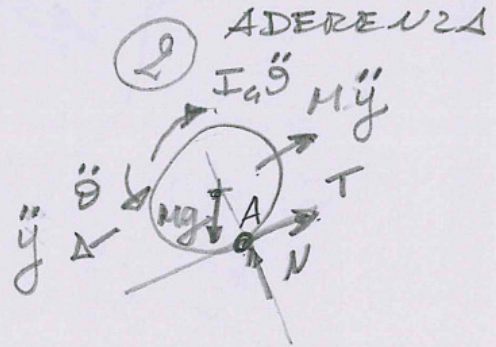
$$\ddot{x} = g (\sin \alpha - f \cos \alpha)$$

$$\ddot{x} = 9,81 (\sin 45^\circ - 0,1 \cos 45^\circ)$$

$$\ddot{x} = 6,24 \text{ m/s}^2$$

$$L = \frac{1}{2} \ddot{x} t^2 \quad t = 6,73 \text{ s}$$

$$V = \ddot{x} t \quad V = 42,01 \text{ m/s}$$



$$\begin{cases} T - Mg \sin \alpha + M \ddot{y} = 0 \\ N - Mg \cos \alpha = 0 \\ \ddot{y} = \ddot{\theta} r \end{cases}$$

$$\text{A) } Mg \sin \alpha r - M \ddot{y} r - I_G \ddot{\theta} = 0$$

$$Mg \sin \alpha r - M \ddot{\theta} r^2 - \frac{MR^2}{2} \ddot{\theta} = 0$$

$$\ddot{\theta} \left( \frac{3}{2} r \right) = g \sin \alpha$$

$$\ddot{\theta} = \frac{2}{3R} g \sin \alpha = 18,50 \frac{\text{rad}}{\text{s}^2}$$

$$\ddot{y} = \ddot{\theta} r = 4,62 \text{ m/s}^2$$

$$L = \frac{1}{2} \ddot{y} t^2 \quad t = 7,82 \text{ s}$$

$$V = \ddot{y} t = 36,16 \text{ m/s}$$

$$\omega = \frac{V}{r} = 144,66 \text{ rad/s}$$





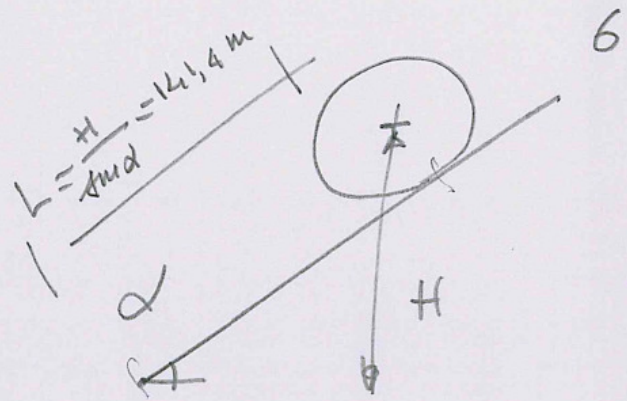
$$M = 100 \text{ kg}$$

$$H = 100 \text{ m}$$

$$\alpha = 45^\circ$$

$$R = 0,25 \text{ m}$$

$$f_{AD} = f = 0,1$$



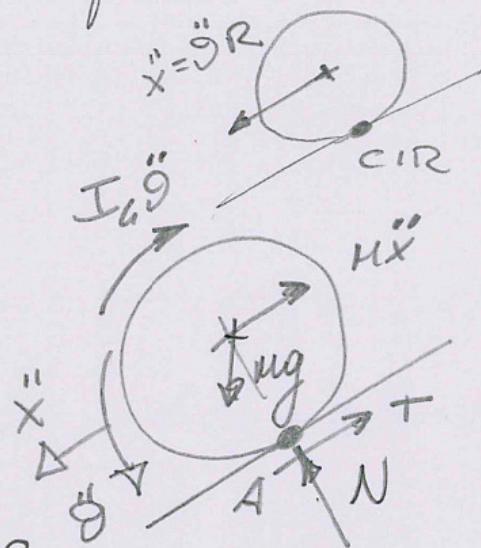
IPOTESI ADERENZA  $T \leq f_{AD} N$

$$1) \ddot{x} = R \ddot{\theta} \text{ DA VERIFICARE } T \leq f_{AD} N$$

$$2) mg \sin \alpha - T - m \ddot{x} = 0$$

$$3) N - mg \cos \alpha = 0$$

$$4) mg \sin \alpha R - m \ddot{x} R - I_G \ddot{\theta} = 0$$



$$mg \sin \alpha R - m R \ddot{\theta} - \frac{m R^2}{2} \ddot{\theta} = 0$$

$$g \sin \alpha - R \ddot{\theta} - \frac{R}{2} \ddot{\theta} = 0$$

$$\ddot{\theta} = \frac{2}{3R} g \sin \alpha = \frac{2}{3} \frac{1}{0,25} 9,81 \sin 45^\circ = 18,50 \frac{\text{rad}}{\text{s}^2}$$

$$N = mg \cos \alpha = 100 \cdot 9,81 \cdot \cos 45^\circ = 693,67 \text{ N}$$

$$T = mg \sin \alpha - m \ddot{\theta} R = m (g \sin \alpha - \ddot{\theta} R) = 231,17 \text{ N}$$

$$\frac{T}{N} \leq f_{AD}; \frac{m (g \sin \alpha - \ddot{\theta} R)}{mg \cos \alpha} = \frac{g \sin \alpha - \ddot{\theta} R}{g \cos \alpha} = 0,33$$



$T, \ddot{x}, \nu, \ddot{\theta}$

$$\rightarrow mg \sin \alpha - T - m\ddot{x} = 0$$

$$\rightarrow \nu - mg \cos \alpha = 0$$

$$\rightarrow mg \sin \alpha R - m\ddot{x}R - I_G \ddot{\theta} = 0$$

$$\rightarrow T = f \nu$$

$$\cancel{mg \sin \alpha R} - \cancel{m\ddot{x}R} - \frac{mR^2}{2} \ddot{\theta} = 0$$

$$g \sin \alpha - \ddot{x} - \frac{R}{2} \ddot{\theta} = 0$$

$$mg \sin \alpha - f mg \cos \alpha - m\ddot{x} = 0$$

$$\ddot{x} = g (\sin \alpha - f \cos \alpha) = 6,24 \text{ m/s}^2$$

$$\ddot{\theta} = \frac{g}{R} (g \sin \alpha - \ddot{x}) = 5,55 \frac{\text{rad}}{\text{s}^2}$$

$$L = \frac{1}{2} \ddot{x} t^2 \quad t = \sqrt{\frac{2L}{\ddot{x}}} = 6,73 \text{ s}$$

$$v = \ddot{x} t \quad v = 42 \text{ m/s}$$

$$\omega = \ddot{\theta} t \quad \omega = 37,35 \text{ rad/s}$$

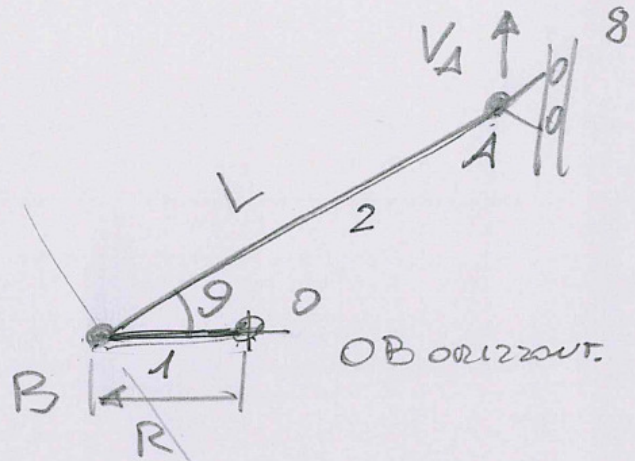
$$* L_i = T \cdot \Delta S$$

$$\Delta S = L - R \cdot \theta$$

$$\theta = \frac{1}{2} \ddot{\theta} t^2 = \frac{1}{2} 5,55 \cdot (6,73)^2 =$$



$\vartheta = 30^\circ$   
 $V_A = 1 \text{ m/s} \cos t$   
 $R = 0,2 \text{ m}$   
 $L = 0,8 \text{ m}$



$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

$\omega_1 R ?$	$1 \text{ m/s}$	$\omega_2 L ?$	$M$
$\perp OB$	$\parallel y$	$\perp AB$	$D$
$?$	$\uparrow$	$?$	$\checkmark$

$\vec{V}_A = \vec{V}_B$

$\omega_2 = 0 \quad V_B = V_A = 1 \text{ m/s} \Rightarrow \omega_1 = \frac{V_B}{R} = \frac{1}{0,2} = 5 \frac{\text{rad}}{\text{s}}$

$a_{Bn} + a_{Bt} = a_A + a_{B/A n} + a_{B/A t}$

$\omega_1^2 R$ $5 \text{ m/s}^2$	$\dot{\omega}_1 R ?$	$0$	$\omega_2^2 L$ $0$	$\dot{\omega}_2 L ?$	$M$
$\parallel OB$	$\perp OB$	$\checkmark$	$\checkmark$	$\perp AB$	$D$
$B \rightarrow 0$	$?$	$\checkmark$	$\checkmark$	$?$	$\checkmark$

$a_{B/A t} = \frac{a_{Bn}}{\sin \vartheta} = 10 \text{ m/s}^2$

$a_{Bt} = \frac{a_{Bn}}{\tan \vartheta} = 8,66 \text{ m/s}^2$

$\dot{\omega}_1 = a_{Bt} / R = 43,30 \frac{\text{rad}}{\text{s}^2} (+\vec{k})$

$\dot{\omega}_2 = \frac{a_{B/A t}}{L} = 12,5 \frac{\text{rad}}{\text{s}^2} (+\vec{k})$

